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MODELING OF COMBAT INTERACTION

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1.0 INTRODUCTION

By combat interaction, is meant the interaction between two opposed forces, Blue and Red, performing functions of FIRE, MANEUVER, and INTELLIGENCE, guided by COMMAND, and supported by elements performing the functions of SUPPLY, TRANSPORTATION, MAINTENANCE, CONSTRUCTION, and SIGNAL. For relative simplicity this paper will discuss only FIRE, MANEUVER, and COMMAND.

2.0 MODELING OF FIRE

2.1 FIRE BY ELEMENTS:

The basic fire event is the discharge of a single round by an element R_j against another element B_i (or vice versa). It is assumed there is a probability of kill, p_{B_i,R_j}, which is a single-valued function of the configuration of R_j (which establishes accuracy and lethality), the configuration of B_i (which establishes basic vulnerability), and the state $\upsilon_{B_i}(t)$ which establishes range from the weapon (affecting accuracy), cover (due to terrain location) and activity B_i.

Since it is impractical to model shot by shot we define a risk $\left(\Phi_{B_i R_j}\right)$ of kill of B_i by R_j in any unit time.

$$\Phi_{B_i R_j} = r_{F,R_j} \ln\left(1 - p_{B_i,R_j}\right)$$

where $r_{F.R_j}$ is some standardized maximum rate of fire by R_j, which depends on its configuration.

Then the probability that Blue will be killed in an interval t_1 to t_2 (assuming only R_j is firing at full rate and Blue is not answering fire) is

$$P_{B_i}\Big]_{t_1}^{t_2} = 1 - \exp\left[\int_{t_1}^{t_2} \Phi_{B_i,R_j}\, dt\right]$$

This formulation ignores, of course, the discrete nature of individual firings - fractional "rounds" are counted.

2.2 FIRE BY UNITS;

The Φ-functions are additive, if more than one Red weapon fires at B_i, and B_i does not shoot back

$$P_{B_i}\Big]_{t_1}^{t_2} = 1 - \exp\left[\int_{t_1}^{t_2} \sum_j \Phi_{B_i,R_j}\, dt\right]$$

If a number of Red weapons are firing at a number (N_B) of unanswering Blue targets, the kill probability of each Blue depends on the priority (or "value") Red places on that particular target. If all are valued equally

$$P_{B_i}\Big]_{t_1}^{t_2} = 1 - \exp\left[\int_{t_1}^{t_2} \frac{\left(\sum_j \Phi_{B_i,R_j}\right)}{N_B}\, dt\right]$$

If Blue is answering fire, the problem becomes more complex, in that the hazard to B_i from any weapon R_j is contingent on that weapon still being alive at the time under consideration.

$$P_{B_i}\Big]_{t}^{t+\Delta t} = \exp\left[\int_{t_1}^{t} \Phi_{B_i}\, dt\right]\left[1 - \exp\left\{\int_{t}^{t+\Delta t} \sum_j \Phi_{B_i,R_j} \circ \exp\left(\int_{t_1}^{t} \Phi_{R_j}\, dt\right) dt\right\}\right]$$

2.3 DIRECTIVE INFORMATION:

The preceding discussions have assumed that the weapons fire at standard rate r_F throughout the interval being computed. Also it has not treated the problem of target selection; to fire in a uniform or random fashion against all available targets is not a feature of any suitable tactical plan.

To take account of fire direction, we introduce the concept of a three-dimensional array $\Lambda_{F,R}$. The elements of the array are labeled by i, j, and ℓ designating target, weapon and time interval Δt, respectively. The element values are in terms of τ_F, the fraction of r_F (including zero) to be applied by weapon R_j against target B_i in time interval Δt_ℓ. Symbolized as $\tau_{F,i,j,\ell}$.

This array $\Lambda_{F,R}$ represents the Red fire course of action. A Red fire plan, an intended course of action which may change during execution, would be represented by $\hat{\Lambda}_{F,R}\Big]_{t_1}^{t_2}$.

$\Lambda_{F,R}$ results from decisions taken at various times and at various levels of command. We represent the final decision, which leads to actual firing of the weapon, as being taken by the element R_j. The decision is based on two pieces of directive information V_{B_i} and $V_{S,R_j,(V)}$, which are developed and passed down the chain of command.[1] They may also be affected by the motivation of R_j (or intervening commanders).[2]

The decision establishes $\tau_{i,j,\ell}$ by:

$$P_{B_i}\Big]_{\Delta t_\ell} V_{B_i}\Big]_{t_2} = \tau_{i,j,\ell}\left(r_{F,R_j}\right)\Delta t_\ell \circ V_{S,R_j,(V)}$$

P_{B_i} is of course a function of $\tau_{i,j,\ell}$, because

$$P_{B_i}\Big]_{\Delta t_\ell} = 1 - \exp\left[\int_{\Delta t_\ell} \tau_{i,j,\ell}\ \Phi_{B_i,R_j}\ dt\right]$$

[1] Value of Blue elements V_{B_i} and value placed on Class V (ammunition) Supply to Red elements $V_{S,R_j,(V)}$ is discussed in Chapter 5, *The Anatomy of Combat,* RJVolluz & RMVolluz, 1996.

[2] Chapter 3, *The Anatomy of Combat,* RJVolluz & RMVolluz, 1996. www.TheAnatomyOfCombat.com

2.4 COORDINATION OF FIRE:

If several Red weapons are to fire at B_i during the interval Δt_ℓ , $\Lambda_{F,R}$ should be adjusted so that

$$P_{B_i}\Big]_{\Delta t_\ell} V_{B_i}\Big]_{t_2} = \sum_j \tau_{i,j,\ell}\left(r_{F,R_j}\right)\Delta t_\ell \circ V_{S,R_j,(V)}$$

The directive variables furnished to each of the j weapons which may fire should be adjusted to produce this result. For instance, weapons which are not to fire at all would be assigned $V_{B_i} = 0$.

This fire coordination is actually a part of the general function of Command. It deserves special attention because of the criticality of weapon response times. R_j is represented as always deciding based on the latest values of V_{B_i} and $V_{S,(V)}$ which it has received; a dynamic tactical situation will require rapid change of these values through a chain of Intelligence-Command-Signal.

2.5 AREA-FIRE WEAPONS:

Higher-caliber (artillery) weapons are typically used for the attack of collections of elemental targets. This phenomenon is caused by Λ_{F,R_j}. A 155 mm round can certainly be lethal against a single infantryman, but it will normally fail the value test described above. The test must become:

$$\sum_i P_{B_i}\Bigg]_{\Delta t_\ell} V_{B_i}\Big]_{t_2} = \tau_{i,j,\ell}\left(r_{F,R_j}\right)\Delta t_\ell \circ V_{S,R_j,(V)} \quad .$$

In effect the weapon must wait until a qualifying collection of elements is identified by the Intelligence function.

3.0 MODELING OF MANEUVER

By maneuver, we imply that positioning and movement of combat elements which is intended to:

1) Lead to occupation or control of some politically-selected terrain objective.

2) Dispose firepower in such a way as to gain in efficiency or reduce the efficiency of enemy firepower.

3.1 MANEUVER BY ELEMENTS

The basic maneuver event is the movement by an element B_i (or R_j) from one terrain "feature" to another along a "route". We define features as being locations such that an element stationary at the feature is less vulnerable than the same element moving on any of the routes leaving the feature.[3] We define no more than one route connecting any pair of features - the fastest one. Routes are characterized by trafficability, which together with the mobility of the element determines the period of movement $\Delta t_{M,B_i}$. The elemental maneuver can then be characterized by $\left(\vec{m}_{B_i}\ , \Delta t_{M,B_i}\right)$.

3.2 MOVEMENT BY UNITS

Movement by a unit is of course just a collectivized view of the elements comprising the unit. It can be regarded as a movement of the center of mass, a change of the shape (external perimeter) of the unit, and a change of the internal arrangement ("formation") of the unit. All of these unit movements are of tactical significance. For the time being, however, we shall concentrated on the "advance" of the unit in a direction $\vec{i}$, characterized by:

$$\vec{M} = \sum_i \vec{m}_{B_i} \circ \vec{i}$$

3.3 DIRECTIVE INFORMATION

We postulate a $\Lambda_{M,B}$ which is analogous to $\Lambda_{F,R}$. It is a two dimensional array with elements $\vec{m}_{i,\ell}$ and $\Delta t_\ell = \Delta t_M$ the time required for the movement $\vec{m}_{i,\ell}$ by B_i .

However, again this is description of a <u>course</u> of <u>action</u>, which is usually guided by some plan $\left.\hat{\Lambda}_{M,B}\right]_{t_1}^{t_2}$, but in execution depends on the decisions made by the elements B_i. B_i moves to the k^{th} feature if and when -

$$Q_k\left(t_2 - \Delta t_M - t\right) \geq \left.p_{B_i}\right]_{\Delta t_M} \circ \left.V_{B_i}\right]_{t_2}$$

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[3] Obviously this definition is dependent on the type-element being discussed - there are less features for tanks than for riflemen.

Since p_{B_i} depends on Φ_{B_i}, the movement may be "suppressed" by Red Fire Potential - the element waits until Φ_{B_i} has been altered sufficiently to make the move profitable. Thus the pattern of movement by any element in high risk conditions will be one of dashes between features interspersed with periods of waiting while the "fire-fight" goes on.

3.4 COORDINATION OF MOVEMENT

If several Blue elements move simultaneously, Red is presented suddenly with a number of priority targets. Φ_{B_i} must be divided among these targets in some fashion, reducing the risk to each, or at least some, of the Blue elements. Blue can advance in the face of risk which is prohibitive for a single element, but the coordination of such movement depends on low-level Status Report/Intelligence-Command-Communication loops. The problem is analogous to the Fire Coordination problem described above; low-level Command-Communication efficiency seriously affects the trade-off between movement and casualties.

The model mechanism for causing such movement is to assign temporarily a very low V_{B_i} to those units which are to move. This may well drop below some floor of V_{B_i} established by individual motivation. In such case only a few individuals may comply.

3.5 INTERACTION OF FIRE AND MANEUVER

We now postulated four arrays describing opposing courses of action $\Lambda_{F,B}$; $\Lambda_{F,R}$; $\Lambda_{M,B}$; and $\Lambda_{M,R}$. These constitute the directive variable input for analysis of a Fire and Maneuver problem. The capability input is described by Φ_B , Φ_R , and sets of $\left(\vec{m}_{B_i}, \Delta t_M\right)$, $\left(\vec{m}_{R_j}, \Delta t_M\right)$ specifying the movement potentials of the elements involved.

A static fire-fight between a Blue force and a Red force during a time interval t_1 to t_2 can be considered as a sequence of kill events occurring probabilistically. Each such event alters the inventory of elements; the battle progresses from a defined initial state at t_1 to some final state at t_2. If one possesses the necessary capability information the probabilities of transition from the initial state to each possible final state can be computed as a Markov chain. Alternatively, one can use expected values in lieu of probability to derive the fundamental Lanchester equations. In making such an analysis one has assumed some pattern of fire $\left(\Lambda_F\right)$ by each side - typically fire at a constant rate throughout the interval, probably without prioritization among the available targets. The probability distribution for end states is a function of the fire capability only. When a discriminating Λ_F is introduced for one or both sides, the problem is changed. Fire is delivered only in accordance with the test (for Red)[4]

$$P_{B_i}\Big]_{\Delta t_\ell} V_{B_i}\Big]_{t_2} \geq \tau_{i,j,\ell}\left(\Lambda_{F,R}\right)\Delta t_\ell \circ V_{S,R_j,(V)} \qquad (\tau \leq 1)$$

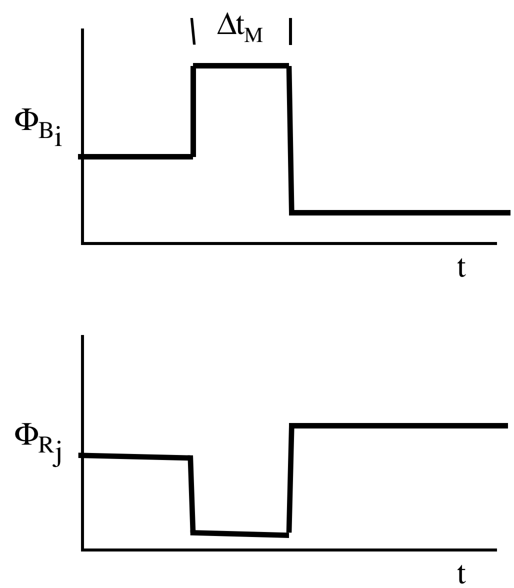
Assuming that the targets do not move and thus change their vulnerability, and that Red Command does not alter the V's, the fire-fight may end with some of the targets untouched. The probability distribution for end states is altered - it has become a function of the directive variables as well as the capability variables.

In battle an end state is actually reached only when one side gives up because it finds its potential for achieving objectives insufficient as compared to the expected cost. If the fire-fight does come to an end, because of lack of profitable targets, without a suitable end state having been reached, either side (or both) can alter the equation in one of two ways:

1) Reduce $V_{S,B_i(V)}$

2) Maneuver to alter Φ_B and/or Φ_R.

The reduction in Item 1), which implies an increased supply of ammunition, can be analyzed without introduction of Λ_M. It simply gives another Lanchester-like solution.



Sketch 1

[4] A corresponding consideration probably holds for Blue.

Maneuver, however, changes the analysis by changing the patterns of Φ_B and Φ_R . In a typical maneuver for tactical advantage B_i may incur a greater risk during the period of movement than by staying in place, but on arriving at the next terrain feature may be at less risk. Correspondingly, the risk to R_j may lessen, or even go to zero, while Blue is moving and then increase if Blue arrives at his destination. A pre-established pattern of such movements will alter the p.d.f. for end states; it becomes a function of Δt_M and the changes in Φ due to movement. Note that if R_j has a characteristic response time (Δt_R) to the altered situation of B_i, the higher risk is incurred only for the period $\Delta t_M - \Delta t_R$. Δt_R is composed of an intelligence lag Δt_I, a command lag Δt_D, and possibly one or more communications lags Δt_X. In the case of higher-caliber indirect-fire weapons , Δt_R ,may be greater than Δt_M .

B_i is represented as moving if and only if:

$$Q_k\left(t_2 - \Delta t_M - t\right) \geq \left(1 - \exp\left[\int_t^{t_2} \Phi_{B_i} dt\right]\right) \circ V_{B_i}\Big]_{t_2}$$

Thus the outcome becomes a function of the assigned Q and V_{B_i} as well as the previously discussed variables.

3.6 MODELING OF COMMAND

For the moment suppose that N_B Blue elements face N_R Red elements. These elements may be of differing kinds (a "force mix") having defined characteristics of vulnerability, firepower and mobility. They interact on terrain consisting of features and routes and described in terms of cover and trafficability. Suppose the two sides are each assigned an unchanging set of directive variables.

The conflict will proceed as follows:

1) Each side will immediately attempt the movements justified in terms of Q_k^B and $V_{B_i}^B$ for Blue, or Q_k^R and $V_{R_j}^R$ for Red.

2) After these are completed a fire-fight will ensue. When casualties occur, further moves may become justified as the casualties cause a drop in Φ.

3) Eventually, however, the fire-fight may die away because of the test involving $V_{B_i}^R$ and $V_{S,R_j(V)}^R$ for Red or $V_{R_j}^B$ and $V_{S,B_i(V)}^B$ for Blue.

If one side must stop moving and/or firing because of directive information, and the other is still active, the first side must change orders in order to respond. If both sides are dormant, either side may take the initiative by changing orders.

It is possible to change orders across the board, for instance by reducing $V_{B_i}^B$ to induce an advance, or reducing $V_{S,B_i(V)}^B$ to cause a renewal of firing. However, this may require permission from higher HQ since it involves casualty and expenditure rates not originally contemplated.

More typically, the Commander must develop a tactical scheme which assigns varying values of V_{B_i} averaging to that prescribed by higher headquarters, and also varying values or $V_{S,B_i(V)}$, averaging as prescribed. An example is that of a "base of fire" to pin down a portion of the enemy forces while a maneuver unit attempts to advance.

In developing a tactical scheme the Commander must:

1) Allocate resources,

2) Establish zones and axes of movement,

3) Assign directive variables to subordinate echelons.

Typically the Commander will consider several tactical schemes, compare each of them to each of several tactical schemes open to the enemy, and select that one which appears calculated to give the best results.

We identify as "Planning" that process in which a command element constructs alternate courses of action for his own side and for the opponent. To construct a course of action he must allocate resources to two or more subordinate commanders, assign zones of action, and establish coordinating times for the attainment of objectives.

Having completed the Planning the command element must go into a process of Estimation. Here he takes each specific pair of opposed courses of action and estimates what the result will be. The result must be expressed in terms of ground gained or held and of casualties to each side; these are then weighted by the directive variables furnished by higher headquarters in order to arrive at the overall worth ("Combat Potential") of that pair of courses of action.

The command element then performs a function of Decision, usually by selecting the maximin, but possibly exercising Generalship or taking advantage of Intelligence regarding the enemy's intentions.

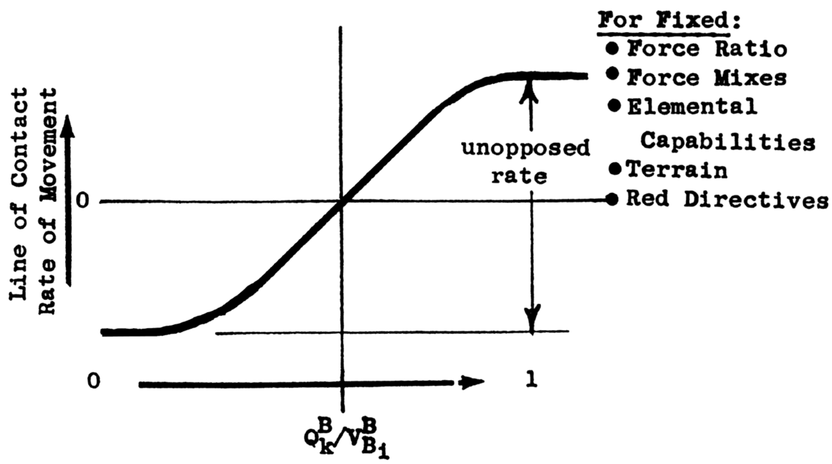
Having made his Decision the command element issues Orders to his subordinate commanders; these are a set $\{........., Q_k(t)\ .\ ....\}$, $\left\{........., V_{B_i}\right]_{t_2}\ .\ ....\Big\}$, $\left\{........., V_{R_j}\right]_{t_2}\ .\ ....\Big\}$, and $\{........., V_{S,B_i(V)}\ .\ ....\}$. The subordinate commanders are then in condition to repeat the process.

Now suppose we hypothesize that movement and casualties are functions of force ratios, force mixes, elemental capabilities, terrain and directive variables issued by both sides.

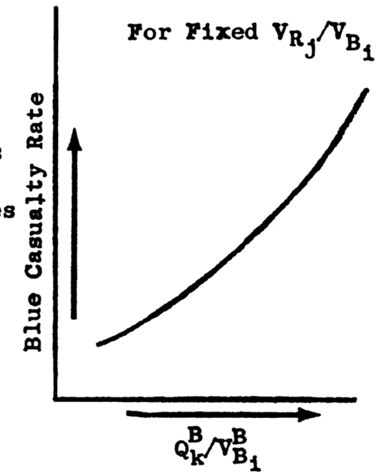
We may picture a command element as having available a set of curves similar to the above which make p.d.f. for outcome in any sector a function of the directive variables he assigns. The actual outcomes for his total force becomes also a function of his command skill - the accuracy of his estimates.

The commander must issue his orders for some time in the future (t_1), in order to allow adequate planning and decision times for the subordinate echelons. This requires projection from the situation as reported at the beginning of plan formulation to a situation expected to obtain at t_1. This is not difficult for a static situation, but the plan when formulated may be upset by enemy action before execution begins. For this reason higher headquarters, who must plan for a t_1 at least 2 to 3 days in the future, normally prepare a large number of contingent plans. Since each of these is considerable effort they are designed to cover a considerable period $(t_1 \text{ to } t_2)$ - week or two at theater level.

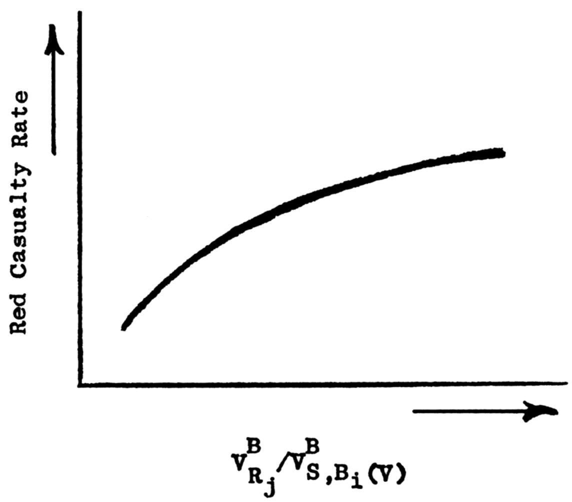
Note [5]



Sketch 2



Sketch 3



Sketch 4

Lower headquarters may be able to fit several planning cycles within the duration of the theater plan - their plans are designed to cover shorter periods, with less contingency planning. A division, for example, will execute a theater plan in 1 to 3-day phases, assigning intermediate objectives en route to the theater objective.

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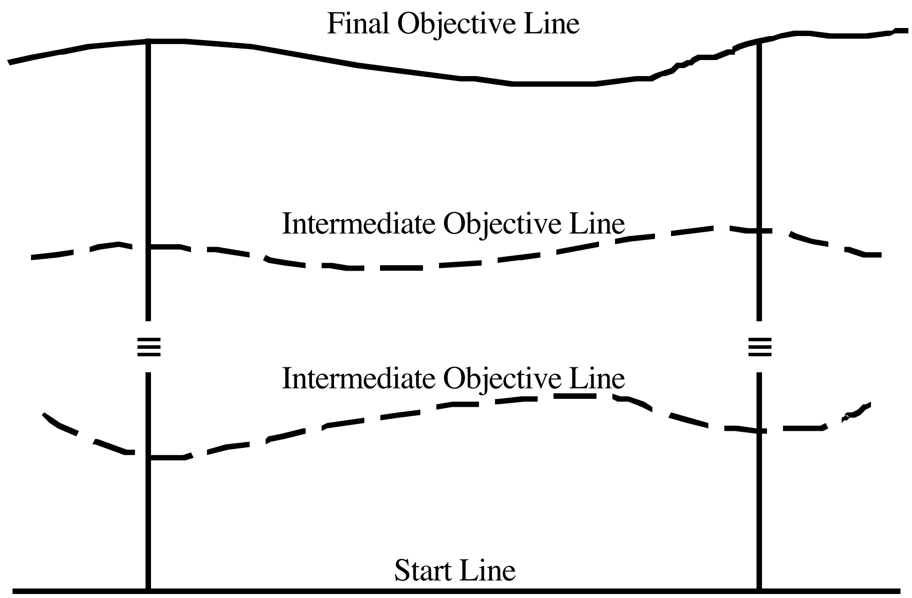
5 Note: Blue casualties are also dependent on $V^{B}_{R_j}\Big|_{t_2}$ if this leads to assignment of high Q_T for the purpose of closing with the enemy.



Sketch 5

When the division commander (for instance) receives the corps plan, he is assigned $\left\{\ldots,Q_k^B(t),\ldots\right\}$ for a final objective, $\left\{\ldots,\left.V_{R_j}^B\right]_{t_2},\ldots\right\}$, $\left\{\ldots,\left.V_{B_i}^B\right]_{t_2},\ldots\right\}$, and $\left\{\ldots,V_{S,B_i(V)},\ldots\right\}$. This establishes final "trade-offs" of casualties and ammunition for enemy casualties and terrain objectives. However, as previously noted, he may assign values differently to his regimental commanders, provided the end result is expected to conform to the corps directive. In addition, the trade-offs are not necessarily at a constant rate during the period. He may plan, for instance, for initial hard fighting with high casualties, in the expectation that this will lead to a breakthrough and later phases will feature rapid advance and lower casualties.