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ANALYSIS OF INTERDICTION ESCORT PROBLEM

HCBrown

1. This description is in terms of the "interdiction escort" problem.
2. Two central new models are needed - the SAM intercept model (SIM) and the air-to-air model (A/AM). Both involve defense against aerial vehicles, and they differ in only one respect. In SIM the hostile vehicles are SAM's which fly a predictable trajectory based on some guidance law. A/AM then differs from SIM only in the necessity for a Red decision routine, and the interaction of that with a Blue decision routine. The Blue decision routine is required for both models.
3. The required models, and their interfaces, are represented schematically in Figure 1. They should each be capable of independent operation whenever the interface is replaced by arbitrary or previously-calculated information. Under present plans, development of the p_i model is not contemplated, so the interface here will be accomplished for the time being by arbitrary input.
4. The SIM model takes input (to be described later) from various sources and performs the following: From a scenario input of inventory, missions (directive information) and initial state¹ (v_1) of the system, calculate the probability distribution for various final states $\Pr[v_2]$. To this end it must have routines which can:
 - a) Calculate state of the system at any time,

¹ v always refers to the "state" - position, vector velocity, orientation, and vector acceleration of some or all elements in the problem. It is subscripted as follows: $v_{a,b}$ where a identifies the time, b the elements described. If b is unspecified, all elements are included. Thus v_1 means the state of all elements at t_1 the beginning of the analysis. v_{L,R_j} means the state of specified Red element (SAM) at t_L the time of its launch. t_2 is the end of the analysis, defined by arrival of some final state where no further interaction is possible (e.g., Blue penetration force leaves defended area).

- b) Project alternate courses of action for Blue
- c) Calculate objective function values and select a course of action,
- d) Develop a "time-line" and sequence events on it.

5. Interface A. SIM interacts with the Red fire doctrine model (FIDO) across interface A. FIDO, which in turn interacts with MSGM (Multi-Site Geometry Model), models the reaction of a Red SAM site to a penetration of its protected zone by a set of Blue airborne targets between times t_1 and t_2 . FIDO has only one input to SIM - the time(s) of launch of Red missile(s). Its selection of these times is based on the state and capabilities of the Blue vehicles at various times. A Blue anti-missile capability alters p_D , the probability of success of the SAM, and therefore it alters the site's optimum response. $\Pr[v_2]$ the probability distribution for various final states implies p_D (a Blue target is either destroyed or not).

The I/O ² across Interface A is $t_2, v_{L,R}, \Pr[v_2]$. FIDO has one present limitation - it manages one site only without reference to possible overlapping capabilities of other sites. This is to be corrected, whether through FIDO or MSGM is undecided.

6. Interface B. The interface between the SAM Intercept model and p_K model (whether the latter is for gun, missile, or advanced weapon) is a "single-shot" probability of kill. "Single-shot" does not necessarily mean single-round - it implies a quantity of fire delivered against one target in one uninterrupted (and usually brief) time period. For a missile system this may be two or three launched simultaneously and following essentially the same track; for a gun-system it may be a volley of several rounds; for the advanced weapon system it is characterized by a power output for a specified "dwell time" ΔT_F .

7. The p_K model must be able to compute p_K as a function of several independent variables, discussed in the next paragraph. The p_K model can then be used to compute a mapping of p_K on these variables, which becomes a tabular input to the SAM Intercept Model. Alternatively, the p_K model may possibly become a sub-routine of the SAM Intercept Model, computing p_K when furnished values of the independent variables.

² Input/Output

8. For the advanced weapon the independent variables establishing p_K are as follows:
- a) $B_{i(E)}$ the identification of the "escort" aircraft which carries the defensive weapon. This identification is understood to include configuration which establishes the performance variables of the weapon system-power, aperture, pointing accuracy, etc. These latter variables should be used internally with the p_K model and its sub models, so that for I/O with SIM or other models, only $B_{i(E)}$ need be specified.
 - b) R_j - the identification of the target, in this case a surface-to-air missile. This identification also includes configuration, which establishes the vulnerability characteristics of the target. If the target presents several vulnerable areas, p_K is to be calculated for that one which gives highest p_K when used as an aim point. This is not necessarily the "softest" point; if the softest point is very small aiming inaccuracy may lead to selection of the "next softest". Note that "configuration" includes "hardening" or other measures to reduce vulnerability.
 - c) v_{F,R_j} - the state of the target at firing time (t_F). The state is expressed in terms of an escort-centered coordinate system, and specifies the following - position (3 variables), vector velocity (3 variables), roll rate (1 variable), attitude with respect to velocity vector (2 variables) with non-zero roll rate, 3 variables without. Axial symmetry reduces the number of variables by 1 in each case).³ This information enables calculation within the p_K model of range, aspect of target, and aiming angle of weapon (there may be limitations mapping into O or reduced p_K for some systems).
 - d) $\vec{v}_{B_{i(E)}}$ - the vector velocity of the escort aircraft. In conjunction with \vec{v}_{R_j} this is used to calculate cross-wind.
 - e) α - the absorption coefficient of the atmosphere.
 - f) Δt_F - length of burst - "dwell time".

³ State also includes vector acceleration. This is of no consequence to the advanced weapon but may be meaningful to a gun model.

9. For a gun system the input to the p_K model would be the same, except that α is unnecessary.

For a defensive missile system two independent variables should be added:

- ρ air density, which affects control limits,
- N_M number of missiles in "single-shot"

Of course the input $B_{i(E)}$ will describe the weapon system differently.

Now if we describe the interface between " p_K model" and "SIM Model" as the collection of variables $(B_{i(E)}, R_j, v_{F,R_j}, \vec{v}_{B_i}, \alpha, \Delta t_F, \rho, N_M)$ and the output p_K the SIM Model can be compatible with three different p_K models, one for each type of system. If properly designed they will not be bothered by information redundant to their problem. The models should be designed to use all of the above information, rather than making simplifying assumptions, which limit the utility of the models.

The symbology must be standardized, at least across interfaces between separate models. The above symbology is recommended.⁴

10. Interface C. The p_I model should be an intelligence model, capable of calculating what information of the enemy is available to a decision element at any given time. In the specific case of SIM, the information of interest to the Blue escort is t_I, v_{I,R_j} for each SAM launched. This information, combined with v_{I,B_i} enables decision as to optimum course of action.

The p_I model would include capabilities of sensors, agents, and processors, and probably other variables not yet defined. Its output would be $\gamma_{R_j}(v_{R_j}(t))$. γ is the risk function for acquisition, defined as $\gamma = \ln(1 - p_D)$, where p_D is probability per unit time that the necessary information is acquired. Then the probability that acquisition has occurred by the time t is $p_I]^t = 1 - \exp\left[-\int_{t_I}^t \gamma dt\right]$. This may be used in convolutions, or to calculate an expected time of acquisition by $p_I]^t = 0.50$.

⁴ Note: v_{F,R_j} and $v_{B_i(E)}$ need further detailing.

As noted above, the lack of this model requires that t_i be covered by assumption.

11. Interface D. The scenario is of course not a model but a description of the problem to be analyzed. When this methodology is used with higher-level models, scenario may input to the Command model, which then transmits input based in part on scenario information.

The scenario must provide these types of information: inventory, directive, environment, and initial state. Inventory information consists of a list and description of the elements entering into the problem. The description is of the element configuration, which establish both vulnerability and performance variables. Of course, it will be desirable to deal with a limited number of type elements, chosen to exhibit the effect of various possible configurations. The elements included in a typical SIM run will be:

- a) Escort aircraft, with anti-missile capability
- b) Strike aircraft (in some cases these may be the same as escort aircraft. This could be either strike aircraft self-defending with guns or missiles or an escort aircraft carrying standoff weapons.)
- c) SAM sites (treated by FIDO)
- d) SAM missiles
- e) Ground Targets for the Strike aircraft

Directive information is necessary for operations of the decision routines. It consists of values assigned to control fire decision (V 's) and values assigned to control maneuver decisions (Q 's). These values may be derived from some total scenario context (higher-level analysis) or may be arbitrarily assigned and parametrically varied.

Since Blue and Red missions and strategies may differ from each other, the directive values may also differ. This is indicated by a superscript. For instance: $V_{B_i(E)}^B$ is the value Blue places on preservation of his escort aircraft; this controls the amount of risk he will assume in his tactics. $V_{B_i(E)}^R$ is the value Red places on destruction of the Blue escort aircraft. This controls the conditions

under which missiles are expended against this aircraft, and the extent to which it is a target preferred over the Blue strike aircraft.

The environment information provided by the scenario is α (absorption coefficient) and ρ (air density). Terrain locations of defense sites and prime targets are also affected by environmental information (masking, cover), however this level of detail is unlikely to be used frequently.

The scenario also specifies the initial state of the system (v_i). The initial state should be such that no interactions between the hostile elements being analyzed has occurred prior to t_i .

12. " p_K " Model. The p_K model takes the input furnished across Interface B (Paras. 6, 7, 8) and produces output in the form of a p_K for the capabilities and conditions specified by input. This must be done for three different weapon types - advanced weapon, guns, and air-to-air missiles. It is not at this point clear whether this can be best done with one generalized model or with three different models.

For the time being we shall talk in terms of a " p_K " model (for advanced weapons), a gun model and an AAM (air-to-air Missile model).

The " p_K " model must consider three separate phenomena:

- 1) The generation and focusing of a beam of energy and its emission from the weapon in a certain selected direction.
- 2) The propagation of this beam through the atmosphere
- 3) The impact of this beam on the target, and the subsequent effects.

The fundamental algorithm proposed for this model is $p_K = \iint \text{Pr}[x,y] \circ L(x,y) dx dy$.

L is the lethality, generated by the lethality model. $\text{Pr}[x,y]$ is the probability density function for accuracy, generated by the Fire Control Model.

The Fire Control Model is to analyze sources of error inherent in the pointing system which initially places the beam on the target (or misses); holding the beam at the point of impingement is

understood to be a different function, probably performed by a different, feed-back system. The effect of errors in this system is understood to be covered by a "jitter" factor.

Since the Fire Control Model covers only the pointing accuracy, the lethality model must cover all other details of the three phenomena listed above. L is actually a conditional probability of kill. It is the probability that a specified target (R_j) is "killed" (degraded in functional capability) given that a beam of a certain description (derived from the specification of B_i) and of a certain time-duration (Δt_F) is propagated through an atmosphere (described by α) for a certain distance (range, determined from v_{F,R_j} to impact at an arbitrary point (x,y) on the target, which presents itself at a certain aspect (also determined from v_{F,R_j}).

The algorithms by which this determination of L is to be made are left to the designer of the lethality model.

The p_K model makes the final calculation of $p_K = \iint \text{Pr}[x,y] \circ L(x,y) dx dy$. In addition, however, it must determine the optimum aim point. The target in a certain aspect should be considered as having superimposed a mapping of values of L versus x,y the coordinates of the point of impingement. The model should select and report as if that point producing highest p_K were used as aim point. This is not necessarily the point of maximum L . If the latter is small in area, the $\text{Pr}[x,y]$ factor may dictate choice of a larger, though somewhat less vulnerable, aim point.

(Figure 1 follows)

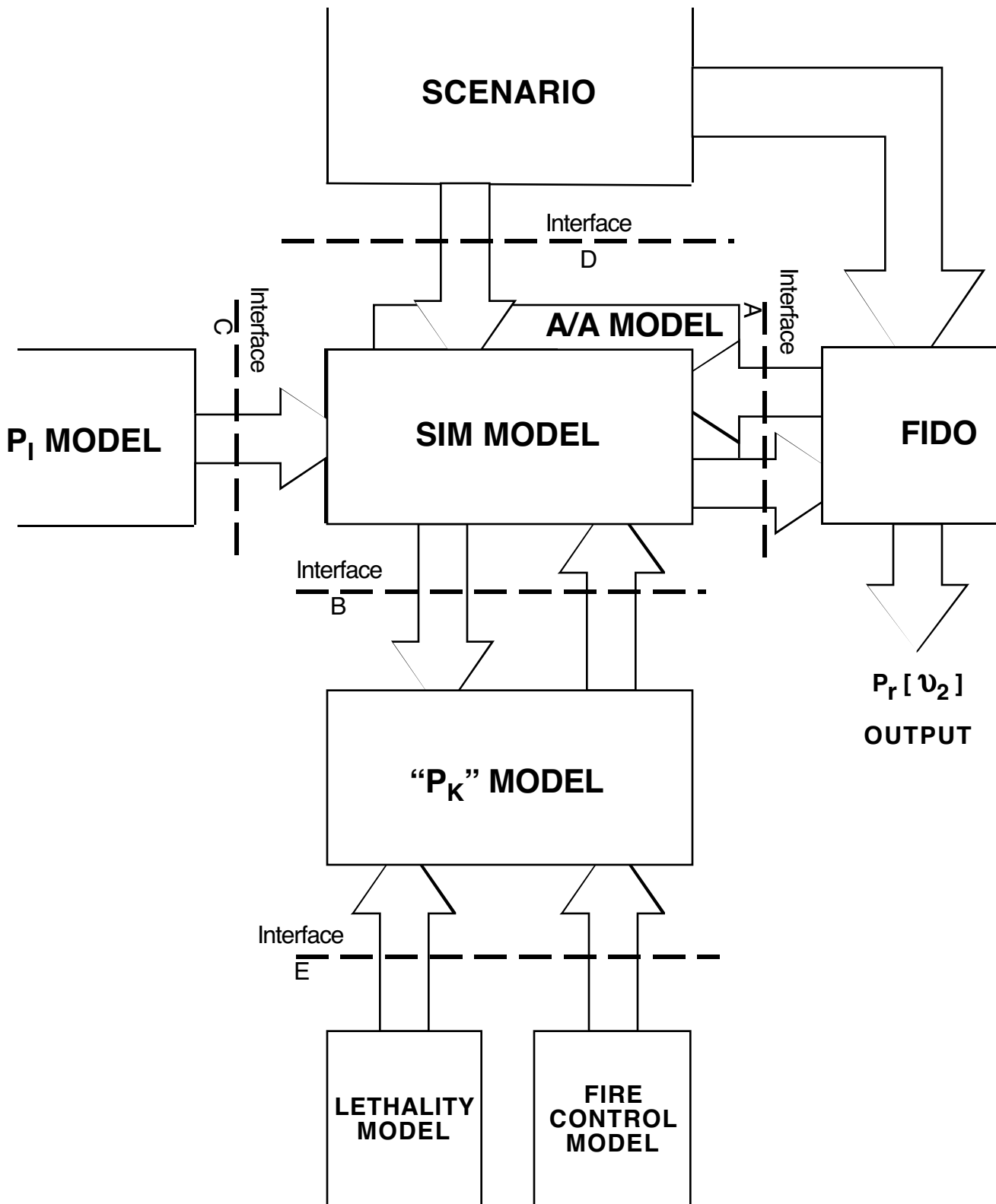


Figure 1