TACWAR # 94
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NOTES ON THE VALUE OF A MILITARY ELEMENT

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The value of a Blue element (V_{B_i}) during an engagement at t_1 is the value of the element's contribution in future engagements planned by the Blue command function. Each future engagement in relation to other future engagements contributes to the attainment of an objective defined by the higher echelon planning functions. This relationship provides a means by which the Blue command estimates the value of success for a particular future engagement. The Blue command uses the estimated success value to determine the element values (V_{B_i}).

The value $V_{B_i}\Big|_{t_1}$ is equal to $V_{B_i}\Big|_{t_2} + \Delta V_{B_i}\Big|_{t_2}^{t_L}$ where ΔV_{B_i} is the contribution of B_i to $\pi_C\Big|_{t_2}^{t_L}$. ΔV_{B_i} is computed in the following examples for a simplified two vs. one engagement for each of two Red fire doctrines.

Integration of the methodology in Attachment (A) with the analysis in Section 2.0, TACWAR #71, provides a solution to the general two vs. one situation which includes the effects of maneuver and occupation of an objective.

The hypothetical engagement is between one Red rifleman and two Blue riflemen, each fixed in position. The Red rifleman has an average kill rate of 1 Blue rifleman per minute and distributes this capability between the two Blue riflemen in such a way that maximum man-days destroyed is achieved. The first Blue rifleman has an average kill rate of 2 Red riflemen per minute, while the second Blue rifleman has an average kill rate of 1 Red rifleman per minute.

If the final outcome is a 90% chance that the Red rifleman is killed, the first Blue rifleman is worth only 2/3 the value of the second Blue rifleman even though the first Blue rifleman has twice the effectiveness of the second.

This analysis assumes Red has apriori knowledge of Blue element values, and as such may be unrealistic. The equation for the relative worth (V_{REL})[1] of the first Blue element to the second is as follows:

$$V_{REL} = \frac{\alpha_{12}(\beta_{11}+\beta_{21})(1-P_W^B)+\alpha_{12}{}^2\left(\frac{\beta_{11}}{\alpha_{11}+\beta_{11}}-P_W^B\right)}{\alpha_{11}(\beta_{11}+\beta_{21})(P_W^B-1)+\alpha_{11}{}^2\left(P_W^B-\frac{\beta_{21}}{\alpha_{12}+\beta_{21}}\right)} \qquad (1)$$

where α_{11} = average kill rate of the Red element vs. Blue element #1

α_{12} = average kill rate of the Red element vs. Blue element #2

β_{11} = average kill rate of Blue element #1 vs. the Red element

β_{21} = average kill rate of Blue element #2 vs. the Red element

P_W^B = probability of Blue win (Red killed)

Another approach is to begin with the assumption that Red distributes his firepower in such a way that maximizes his own chances of survival (no apriori knowledge is required here, since Red is capable of observing the Blue firepower from an ideal[2] vantage point right from the outset).

The equation for V_{REL}[3] is changed to the following:

$$V_{REL} = \frac{P_W^B\, V_{REL}^{'} - V_{REL}^{''}}{1-P_W^B} - 1 \qquad (2)$$

[1] See Attachment (A) for derivation.

[2] Since Red is on the receiving end, the word "ideal" is used in an analytical sense only.

[3] For derivation see Attachment (A)

where

$$P_W^B = \frac{(\beta_{11}+\beta_{21})(\alpha_{11}\beta_{11}+\alpha_{12}\beta_{21}) + \frac{\alpha_{11}{}^2\beta_{21}\beta_{11}}{\alpha_{12}+\beta_{21}} + \frac{\alpha_{12}{}^2\beta_{21}\beta_{11}}{\alpha_{11}+\beta_{11}}}{(\beta_{11}+\beta_{21})(\alpha_{11}\beta_{11}+\alpha_{12}\beta_{21}) + \alpha_{11}{}^2\beta_{11} + \alpha_{12}{}^2\beta_{21}}$$

α's and β's are defined as for Equation (1).

$V_{REL}^{'}$ = Ratio of Red element value to Blue element #2 value.

$V_{REL}^{''}$ = Ratio of combat potential to Blue element #2 value .

Re-working the first example using Equation (2) with the following additional information:

$V_{REL}^{'}$ = 1

$V_{REL}^{''}$ = parameterized

We find values for V_{REL} as shown in Figure 1.

Given estimates of expected combat potential and relative Red element value, the relative worth of the Blue elements are computed from Equation (2).

Figure 1 illustrates the following truth implicit in any battle plan:

> "The value of a combat element is proportional in a negative manner to the expected military worth of the battle within which the element is participating."

The variation of element value with element effectiveness can be easily computed using either Equation (1) or (2).

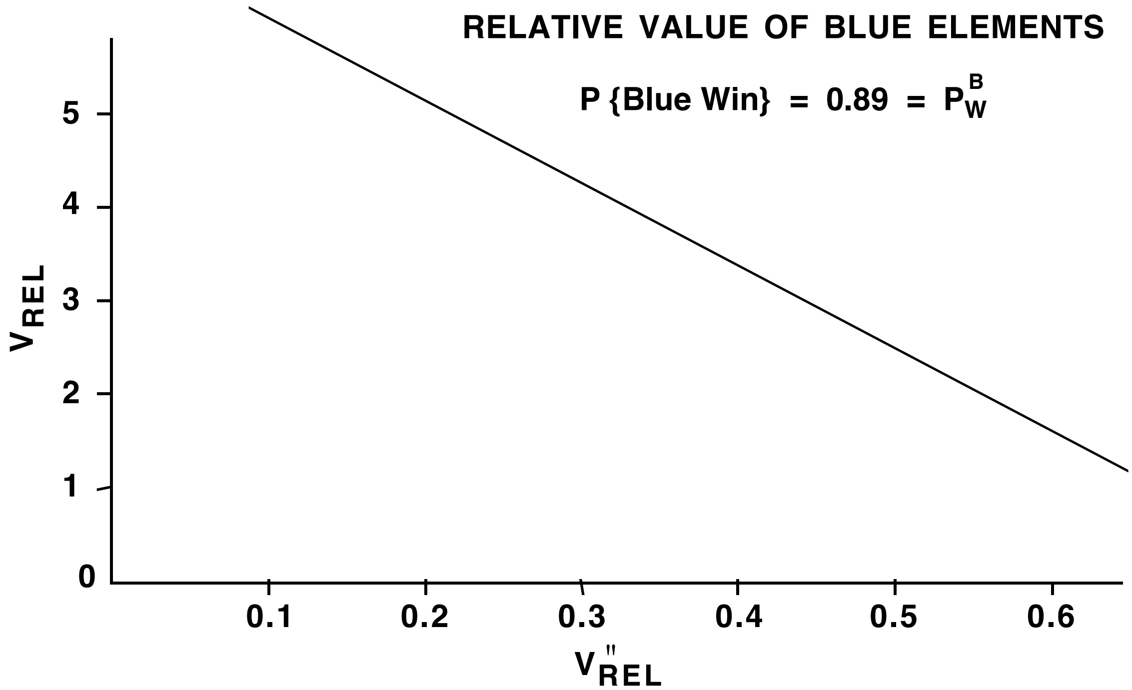
RELATIVE VALUE OF BLUE ELEMENTS
P {Blue Win} = 0.89 = P$_W^B$
5
4
3
2
1
0
V$_{REL}$
0.1
0.2
0.3
0.4
0.5
0.6
V$''_{REL}$

Figure 1

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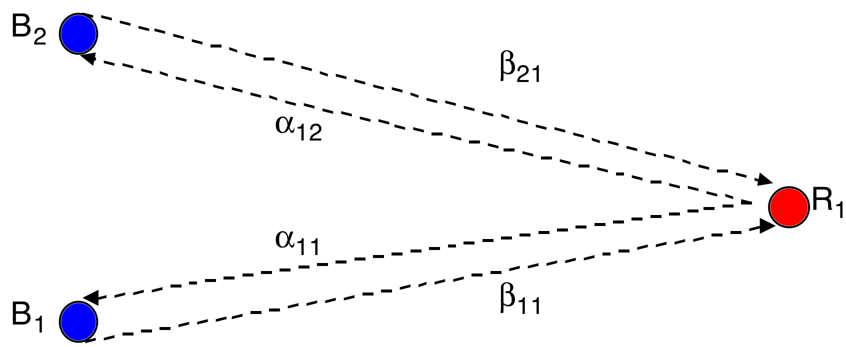
ATTACHMENT A

COMBAT ENGAGEMENT MODEL METHODOLOGY (TWO VERSUS ONE)

Consider an engagement involving two Blue elements and one Red element with attrition capabilities β_{11} , β_{21} , α_{11} , α_{12} as shown in the diagram.



Sketch A - 1

(Note: The α 's and the β 's are the expected number of elements killed per unit of time as derived from the Poisson distribution.)

The state matrix listing all states of existence during the course of the battle is shown below ("1" implies functioning, "0" implies "killed").

State #	Blue #1	Blue #2	Red #1
1	1	1	1
2	0	1	1
3	1	0	1
4	0	0	1
5	1	1	0
6	0	1	0
7	1	0	0

If $P_i(t)$ is defined as the probability of the battle existing in state i at time t, the differential equations for the $P_i(t)$ are

$$\frac{\partial P_1(t)}{\partial t} = -q_1 P_1(t)$$

$$\frac{\partial P_2(t)}{\partial t} = -q_2 P_2(t) + q_{21} P_1(t)$$

$$\frac{\partial P_3(t)}{\partial t} = -q_3 P_3(t) + q_{31} P_1(t)$$

$$\frac{\partial P_4(t)}{\partial t} = q_{42} P_2(t) + q_{43} P_3(t) \tag{1}$$

$$\frac{\partial P_5(t)}{\partial t} = q_{51} P_1(t)$$

$$\frac{\partial P_6(t)}{\partial t} = q_{62} P_2(t)$$

$$\frac{\partial P_7(t)}{\partial t} = q_{73} P_3(t)$$

where

$$q_1 = \beta_{11} + \beta_{21} + \frac{\alpha_{11}{}^2 V_{b_1} + \alpha_{12}{}^2 V_{b_2}}{\alpha_{11} V_{b_1} + \alpha_{12} V_{b_2}}$$

$$q_2 = \alpha_{12} + \beta_{21}$$

$$q_3 = \alpha_{11} + \beta_{11}$$

$$q_{21} = \alpha_{11} + \frac{\alpha_{11} V_{b_1}}{\alpha_{11} V_{b_1} + \alpha_{12} V_{b_2}}$$

$$q_{31} = \alpha_{12} + \frac{\alpha_{12} V_{b_2}}{\alpha_{11} V_{b_1} + \alpha_{12} V_{b_2}} \quad (2)$$

$$q_{42} = \alpha_{12}$$

$$q_{43} = \alpha_{11}$$

$$q_{51} = \beta_{11} + \beta_{21}$$

$$q_{62} = \beta_{21}$$

$$q_{73} = \beta_{11}$$

The equations for q_i and q_{ij} imply that the Red element distributes his fire between b_1 and b_2 in such a way that the probability of Red firing at b_i is equal to $\left(\dfrac{\alpha_{11} V_{b_1}}{\alpha_{11} V_{b_1} + \alpha_{12} V_{b_2}}\right)$.

In words, the Red element fires at the Blue element with a frequency proportional to Red's effectiveness against the Blue element times the value of the Blue element.

The probability that Blue will win the battle, P_W^B, is equal to the sum of P_5, P_6, and P_7. If the value of t becomes large so that $P_1 \approx P_2 \approx P_3 \approx 0$, then P_W^B may be solved from (1) as

$$P_W^B = \frac{1}{q_1}\left[q_{51} + \frac{q_{62}\, q_{21}}{q_2} + \frac{q_{73}\, q_{31}}{q_3}\right] \tag{3}$$

The ratio of Blue element values is defined as

$$V_{REL} = V_{b_1} / V_{b_2}$$

Solving (3) for V_{REL},

$$V_{REL} = \frac{\alpha_{12}(\beta_{11}+\beta_{21})(1-P_W^B) + \alpha_{12}^{\,2}\left(\dfrac{\beta_{11}}{\alpha_{11}+\beta_{11}} - P_W^B\right)}{\alpha_{11}(\beta_{11}+\beta_{21})(P_W^B-1) + \alpha_{11}^{\,2}\left(P_W^B - \dfrac{\beta_{21}}{\alpha_{12}+\beta_{21}}\right)} \tag{4}$$

If Red modifies the fire doctrine to maximize his survivability, Equations (2) are modified as follows

$$q_1 = \beta_{11} + \beta_{21} + \frac{\alpha_{11}{}^2 \beta_{11} + \alpha_{12}{}^2 \beta_{21}}{\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21}}$$

$$q_2 = \alpha_{12} + \beta_{21}$$

$$q_3 = \alpha_{11} + \beta_{11}$$

$$q_{21} = \frac{\alpha_{11}{}^2 \beta_{11}}{\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21}}$$

$$q_{31} = \frac{\alpha_{12}{}^2 \beta_{21}}{\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21}} \quad (5)$$

$$q_{42} = \alpha_{12}$$

$$q_{43} = \alpha_{11}$$

$$q_{51} = \beta_{11} + \beta_{21}$$

$$q_{62} = \beta_{21}$$

$$q_{73} = \beta_{11}$$

Inserting the Equations (5) into Equation (3)

$$P_W^B = \frac{(\beta_{11}+\beta_{21})(\alpha_{11}\beta_{11}+\alpha_{12}\beta_{21}) + \frac{\alpha_{11}{}^2\beta_{21}\beta_{11}}{\alpha_{12}+\beta_{21}} + \frac{\alpha_{12}{}^2\beta_{21}\beta_{11}}{\alpha_{11}+\beta_{11}}}{(\beta_{11}+\beta_{21})(\alpha_{11}\beta_{11}+\alpha_{12}\beta_{21}) + \alpha_{11}{}^2\beta_{11} + \alpha_{12}{}^2\beta_{21}}$$

The combat potential π_C is equal to

$$\pi_C = P_W^B V_R - P_W^R \left(V_{b_1} + V_{b_2}\right) \quad , \quad Q\Delta t \text{ assumed equal to zero}$$

Since $P_W^R = 1 - P_W^B$

$$\frac{V_{b_1}}{V_{b_2}} = \frac{P_W^B \frac{V_R}{V_{b_2}} - \frac{\pi_C}{V_{b_2}}}{1 - P_W^B} - 1$$

let

$$V_{REL} = \frac{V_{b_1}}{V_{b_2}}$$

$$V_{REL}^{'} = \frac{V_R}{V_{b_2}}$$

$$V_{REL}^{''} = \frac{\pi_C}{V_{b_2}}$$

then

$$V_{REL} = \frac{P_W^B V_{REL}^{'} - V_{REL}^{''}}{1 - P_W^B} - 1$$