TACWAR # 71
5 Sept. 72

COMBAT ENGAGEMENT MODEL METHODOLOGY (ONE VS. ONE) - MOD 0

DJGadler

1.0 INTRODUCTION

This paper develops all of the methodology believed to be necessary to evaluate the one-on-one engagement integrating the command function with the engagement analysis. Inputs to the analysis are:

1) Edited Intelligence and Status Reports

2) Completion of Command - Phase I (t_1), Resulting in Alternative C/A's

It is believed that a quantitative solution to this problem will provide the necessary ground-breaking to easily proceed to the higher level planning functions.

Courses of Action (C/A's) referred to herein constitute combination of Fire and Maneuver C/A's with implied Intelligence C/A's, which collectively become a combTW071 Sketch /tw071 Fig 1at course of action. Fire or maneuver C/A's are not considered independently at this level.

2.0 ANALYSIS

Let d_{ij} = distance between terrain features (i,j)

$S_{ij,B}$ = speed of travel between features (i,j) for Blue

$S_{ij,R}$ = speed of travel between features (i,j) for Red

$\Delta t_{i,B}$ = Blue time of stay on feature i

$\Delta t_{i,R}$ = Red time of stay on feature i

$$\text{A Blue C/A} = \left\{k_1, \Delta t_{k_1,B}\ ;\ k_2, \Delta t_{k_2,B}\ ;\ \ldots\ldots\ldots\ldots\ ;\ k_N, \Delta t_{k_N,B}\right\}_B$$

$$\text{A Red C/A} = \left\{m_1, \Delta t_{m_1,R}\ ;\ m_2, \Delta t_{m_2,R}\ ;\ \ldots\ldots\ldots\ldots\ ;\ m_M, \Delta t_{m_M,R}\right\}_R$$

where k_i and m_i are terrain features.

Total E.T. for the Blue C/A is $\sum_{i=1}^{N-1} \frac{d_{k_{i+1},k_i}}{S_{k_{i+1}\,k_i,B}} + \Delta t_{k_i,B}$

and for Red $\sum_{i=1}^{M-1} \frac{d_{m_{i+1},m_i}}{S_{m_{i+1}\,m_i,R}} + \Delta t_{m_i,R}$

The time to arrive at terrain feature k_i or m_i for Blue is

$$t_{k_i,B} = \sum_{i=1}^{j-1} \frac{d_{k_{i+1},k_i}}{S_{k_{i+1}\,k_i,B}} + \Delta t_{k_i,B} \quad , \quad t_{k_1,B} = 0$$

and for Red

$$t_{m_i,R} = \sum_{i=1}^{j-1} \frac{d_{m_{i+1},m_i}}{S_{m_{i+1}\,m_i,R}} + \Delta t_{m_i,R} \quad , \quad t_{m_1,R} = 0$$

The "target" is located somewhere between terrain features or on a terrain feature at some point in time, t . It is assumed that an element which is moving from one terrain feature to another cannot fire effectively at the opponent.

Given the Red element on feature m_ℓ , the range to the target element as a function of time may be approximated by:

$$R_{m_\ell}(t) = \sum_{i+1}^{N-1}\left[\left\{d_{m_\ell k_i} - S_{k_{i+1}k_i,B}\left(\frac{d_{m_\ell k_i} - d_{m_\ell k_{i+1}}}{d_{k_{i+1}k_i}}\right)\left(t - t_{k_i,B} - \Delta t_{k_i,B}\right)\right\}\left\{u\left(t - t_{k_i,B} - \Delta t_{k_i,B}\right) - u\left(t - t_{k_{i+1}B}\right)\right\}\right.$$
$$\left. + d_{m_\ell k_i}\left\{u\left(t - t_{k_i,B}\right) - u\left(t - t_{k_i,B} - \Delta t_{k_i,B}\right)\right\}\right]$$

where $u(x) = \begin{cases} 0 \ , \ x < 0 \\ 1 \ , \ x \geq 0 \end{cases}$

A similar equation applies when the Blue element fires at the Red element.

Equations have been derived to compute the range to target given that the element bringing fire is located on a terrain feature. An important distinction then is whether an element is moving (not firing), or stationary (firing). Define the variable $\zeta_B(\zeta_R)$ such that if

$\zeta_B = 0$, Blue is moving

$\zeta_B = 1$, Blue is stationary

Then $$\zeta_B(t) = \sum_{i=1}^{N-1}\left\{u\left(t - t_{k_i,B}\right) - u\left(t - t_{k_i,B} - \Delta t_{k_i,B}\right)\right\}$$

$$\zeta_R(t) = \sum_{i=1}^{M-1}\left\{u\left(t - t_{m_i,R}\right) - u\left(t - t_{m_i,R} - \Delta t_{m_i,R}\right)\right\}$$

Given average Blue fire rate $v_{F,B}$ and average Red fire rate $v_{F,R}$, the fire rates as a function of time are respectively:

$$v_{F,B}(t) = v_{F,B}\ \zeta_B(t)$$

$$v_{F,R}(t) = v_{F,R}\ \zeta_R(t)$$

These average rates will be applied later in a stochastic fashion to compute the probabilities of success.

If either element is stationary, the terrain feature currently being occupied is

$$\Delta_B = \sum_{i=1}^{N-1} k_i \left\{ u\left(t - t_{k_i,B}\right) - u\left(t - t_{k_i,B} - \Delta t_{k_i,B}\right)\right\} \text{ for Blue , and}$$

$$\Delta_R = \sum_{i=1}^{M-1} m_i \left\{ u\left(t - t_{m_i,R}\right) - u\left(t - t_{m_i,R} - \Delta t_{m_i,R}\right)\right\} \text{ for Red.}$$

The speed of an element as a function of time is

$$S_B(t) = \sum_{i=1}^{N-1} S_{k_i\,k_{i+1},B} \left\{ u\left(t - t_{k_i,B} - \Delta t_{k_i,B}\right) - u\left(t - t_{k_{i+1}B}\right)\right\}$$

$$S_R(t) = \sum_{i=1}^{M-1} S_{m_i\,m_{i+1},R} \left\{ u\left(t - t_{m_i,R} - \Delta t_{m_i,R}\right) - u\left(t - t_{m_{i+1}R}\right)\right\}$$

All the terrain "tools" now exist to compute effectiveness. Each element has a capability defined by the accuracy of the weapon, the vulnerability of the target and the fire rate of the weapon.

Weapon accuracy is expressed as a function of range $\sigma_B(R)$ for Blue and $\sigma_R(R)$ for Red. Further, each element has some vulnerable area $A_{v,B}$ for Blue and $A_{v,R}$ for Red. These quantities are not constant, however, but vary from terrain feature to terrain feature. If we let the $A_{v,B}$ and $A_{v,R}$ represent maximum values, then a reduction factor may be introduced which is a function of terrain features.

Let $C_{v_{ij}}$ be the vulnerability reduction factor due to terrain feature i as viewed from terrain feature j. This factor is independent of whether the element on the feature is Red or Blue.

Let $C_{\sigma,B}(s)$ and $C_{\sigma,R}(s)$ be functions which modify the accuracy $\sigma(R)$. $C_{\sigma}(s)$ is a function of target speed, s , and applies only when the target is moving ($C_{\sigma}(0)=1$).

Now the single shot probability of kill ($P_{k\,ss}$) as a function of time may be computed as follows (assuming that 1) a shot is fired, and 2) the impact is circular normal about the target vulnerable area which is also assumed circular):

$$P_{k\,ss,R}(t)=1-e^{-\frac{1}{2\pi}\frac{A_{v,R}\;C_{v_{\Delta R,\Delta B}}}{\left\{\sigma_B\left(R_{\Delta_B}(t)\right)C_{\sigma,B}(S_R(t))\right\}^2}} \quad , \quad \Delta_B \neq 0$$

$$=0 \quad , \quad \Delta_B = 0$$

The equation for $P_{k\,ss,B}(t)$ is similar.

The average attrition rate, $\beta(t)$ for Blue against Red and $\alpha(t)$ for Red against Blue is:

$$\beta(t)=P_{k\,ss,R}(t)\;v_{F,B}(t)$$

$$\alpha(t)=P_{k\,ss,B}(t)\;v_{F,R}(t)$$

The possible states of existence for the Red and Blue elements are shown in the following table (A "1" implies existence; a "0" implies non-existence):

State	Blue	Red
1	1	1
2	1	0
3	0	1

(Note that simultaneous kill has not been considered. Recognizing that certain situations where the weapon time of light is significant may allow such an occurrence, the probability of the event is still very small. The inclusion of mutual kill in the analysis would unnecessarily complicate the methodology.

Define $P_i(t)$ as the probability of existence in state i at time t .

If we assume a Poisson type distribution of shots fired, we may express the probability of transition from one state to another in time interval $(t\ ,\ t+\Delta t)$ as:

$$P_1(t+\Delta t) \cong P_1(t)\left[1-\alpha(t)\Delta t-\beta(t)\Delta t\right]$$

$$\text{(note } \Delta t^n << 1 \quad , \quad n \geq 2\text{)}$$

$$P_2(t+\Delta t) \cong P_2(t)+P_1(t)\beta(t)\Delta t$$

$$P_3(t+\Delta t) \cong P_3(t)+P_1(t)\alpha(t)\Delta t$$

Letting $\Delta t \to 0$ we have the following set of differential equations:

$$\frac{\partial P_1(t)}{\partial t} = -\left[\alpha(t)+\beta(t)\right] P_1(t)$$

$$\frac{\partial P_2(t)}{\partial t} = \beta(t)\ P_1(t)$$

$$\frac{\partial P_3(t)}{\partial t} = \alpha(t)\ P_1(t)$$

The potential of a particular course of action is expressed as a function of these probabilities, and a set of values assigned to each element and to the terrain objective:

Let Q_I = objective value (men)

V_R = Red element residual value[1] (man-days)

V_B = Blue element residual value (man-days)

---

1 Residual value - includes cost of repair or replacement + situational value until repaired or replaced.

Q_I^B = objective value

V_R^B = Red residual value as viewed by the Blue Command Function

V_B^B = Blue residual value

Q_I^R = objective value

V_R^R = Red residual value as viewed by the Red Command Function

V_B^R = Blue residual value

A particular C/A may now be evaluated by the potential as computed from the following equations:

Let $O_i = i^{th}$ Blue C/A

$\nabla_j = j^{th}$ Red C/A

$\pi_{O_i \nabla_j ,B}$ = Blue potential of the i^{th} Blue C/A and the j^{th} Red course of action.

$\pi_{O_i \nabla_j ,R}$ = Red potential of the j^{th} Red C/A and the i^{th} Blue course of action.

$\pi^B_{O_i \nabla_j ,B}$ = Blue potential as viewed by the Blue command function

$\pi^R_{O_i \nabla_j ,R}$ = Red potential as viewed by the Red command function

We have three planning (game) matrices: One for the Blue command, One for the Red command, and One for the Umpire.

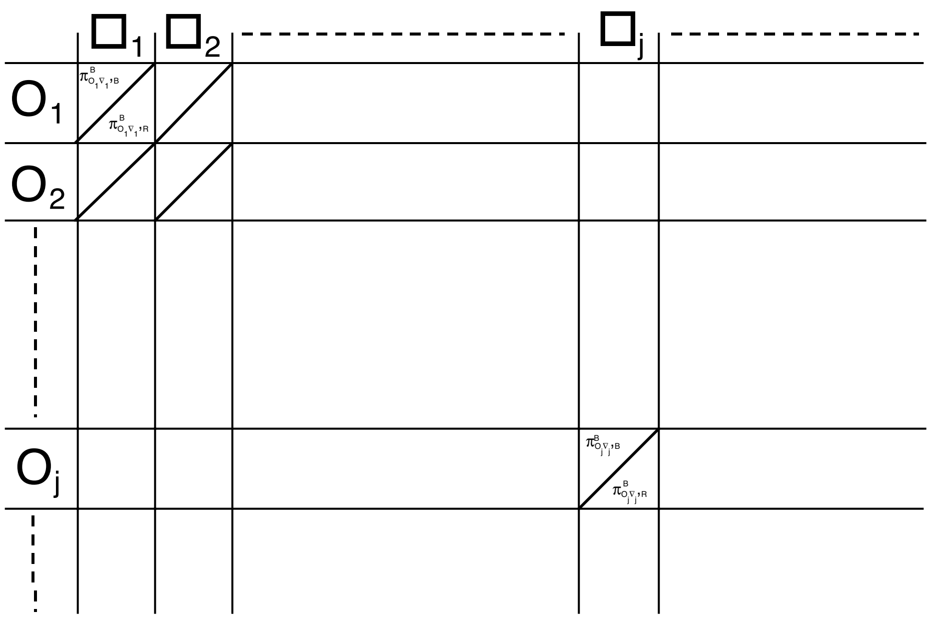
The Umpire computes the potentials for each pair of Red and Blue C/A's as follows. Times $\tau_{2,B}$ for Blue, and $\tau_{2,R}$ for Red are defined the latest times at which the objective has a value to the respective force. The total value of the objective is then equal to its value Q_I times the length of time the element can remain on it before τ_2 .

$$\pi^{B}_{O_i \nabla_j, B} = \left[P_1\left(t_{k_N,B}\right) + P_2\left(t_{k_N,B}\right) \right] Q_I \left[\tau_{2,B} - t_{k_N,B} \right] + P_2\left(\tau_{2,B}\right) V_R - P_3\left(\tau_{2,B}\right) V_B$$

$$\pi_{O_i \nabla_j, R} = \left[P_1\left(t_{m_M,R}\right) + P_3\left(t_{m_M,R}\right) \right] Q_I \left[\tau_{2,R} - t_{m_M,R} \right] + P_3\left(\tau_{2,R}\right) V_B - P_2\left(\tau_{2,R}\right) V_R$$

Similar expressions but with the proper values for Q_I ,V_R , and V_B exist for the Red and Blue command functions.

The Blue command planning (game) matrix has the following form:



Sketch A

As a first cut, we will assume that Blue evaluates only the $\pi^{B}_{O_i \nabla_j, B}$ elements without regard to the $\pi^{B}_{O_i \nabla_j, R}$. Blue command then selects the maxi-min $\pi^{B}_{O_{i^*} \nabla_{j^*}, B}$. Similarly Red selects his maxi-min $\pi^{R}_{O_{i^{**}} \nabla_{j^{**}}, R}$.

The actual potentials as evaluated by the Umpire are:

$$\pi_{O_{i^*} \nabla_{j^{**}}, B} \quad \text{and} \quad \pi_{O_{i^*} \nabla_{j^{**}}, R}$$

The efficiency of command is computed as follows:

Given $\nabla_{j^{**}}$ there exists a $O_{i^{***}}$

such that $\pi_{O_{i^{***}}\nabla_{j^{**}},B} \geq \pi_{O_i\nabla_{j^{**}},B}$, for all i

Then $\eta_{D,B} = \pi_{O_{i^*}\nabla_{j^{**}},B} \big/ \pi_{O_{i^{***}}\nabla_{j^{**}},B}$

Similarly $\eta_{D,R} = \pi_{O_{i^*}\nabla_{j^{**}},R} \big/ \pi_{O_{i^*}\nabla_{j^{***}},R}$

where the η_D 's are the efficiencies of the command functions.

The objective of command is to make the η_D 's as close to one as possible. The attainment of this objective is a function of the inputs to the command function, for if the command had the knowledge of the Umpire, η_D would in fact be equal to one.

3.0 NUMERICAL EXAMPLE

The following problem is presented as an example:

> "Technology has indicated a capability to be able to design a new rifle which will allow the user to fire at an increased rate but with the same accuracy as that obtained while using the existing weapon under slow aimed fire conditions. Should this new weapon be designed and placed in the inventory, and if so, what should the fire rate be?"

The question, when placed in the Anatomy of Combat methodological framework, becomes:

"What is the tactical utility of increased fire rate?"

Equations have been developed to compute the combat potential of the weapon; therefore, the solution will be in terms of a $\Delta\pi_C$ (change in combat potential) where:

$$\Delta\pi_C = \pi_C \text{ (with the new weapon) - } \pi_C \text{ (with the old weapon)}$$

A weapon such as this, being a replacement for an existing weapon, modifies the supply function by requiring more ammunition during a time period. Therefore, the requirement for an overall increase in tactical utility is stated by the following equation:

$$\Delta\left(\frac{\pi_C}{W\Delta t}\right) > 0$$

where, $W\Delta t$ is the supply man-day investment during the time segment Δt.

For simplicity, let $Q_I^B = Q_I^R = Q_I$

$$V_B^B = V_B^R = V_B$$

$$V_R^B = V_R^R = V_R$$

Assume that Red occupies the objective terrain feature and does not leave unless to retreat.

Consider three terrain features:

Δ_1 is the feature occupied by Blue at time $t=0$.

Δ_2 is midway between feature Δ_1 and the objective feature

Δ_3 is the objective feature occupied by Red at time $t=0$

Therefore: $d_{12} = d_{23} = d_{21} = d_{32}$

Let $d_{13} = D$

Then $d_{12} = D/2 = d_{23}$

Now $\Delta t_{1,B} + \Delta t_{2,B} + \text{(travel time)} \leq \tau_{2,B}$

where $\tau_{2,B}$ is the time by which Blue must take the objective.

Let $S_{12,B} = S_{23,B} = S$ (speed of travel)

Then $$\Delta t_{1,B} + \Delta t_{2,B} + \frac{D}{S} \leq \tau_{2,B} \qquad (1)$$

The Red strategy is of course $\{3, \Delta t_{3,R}\}_R$ until such time as a retreat is advantageous.

$\therefore \quad \Delta\tau_{3,R}$ is also a variable.

A retreat will be modeled as a move by Red to an imaginary feature Δ_4 .

By letting $C_{\sigma,B}(S \neq 0) \rightarrow \infty$ we assure that no further hostile action will take place following initiation of the retreat.

Possible Blue C/A's are:

$O_1 = \{1, \Delta t_{1,B} \ ; \ 2, \Delta t_{2,B} \ ; 3\}$ and $O_2 = \{1, \Delta t_{1,B} \ ; 3\}$

however, by letting $\Delta t_{2,B} = 0$

then O_1 becomes O_2

Therefore, we need only consider the one C/A O_1 where $\Delta t_{1,B}$ and $\Delta t_{2,B}$ are variables constrained by Equation (1).

Capability inputs are: $\sigma_B(R)$, $\sigma_R(R)$

$v_{F,B}$, $v_{F,R}$

$C_{\sigma,B}$, $C_{\sigma,R}$

$C_{v_{13}}$, $C_{v_{23}}$, $C_{v_{31}}$, $C_{v_{32}}$

$A_{v,B}$, $A_{v,R}$

The potentials are computed from the following equations:

$t_{3,B}$ = time of arrival on Δ_3

$= \Delta t_{1,B} + \Delta t_{2,B} + D/S$

Then,

$$\pi_{c,B} = \left[P_1(t_{3,B}) + P_2(t_{3,B})\right] Q_I \left[\tau_{2,B} - t_{3,B}\right] + P_2(\tau_{2,B})V_R - P_3(\tau_{2,B})V_B$$

$$\pi_{c,R} = \left[P_1(\Delta t_{3,R}) + P_3(\Delta t_{3,R})\right] Q_I \left[\Delta t_{3,R}\right] + P_3(\Delta t_{3,R})V_B - P_2(\Delta t_{3,R})V_R$$

for particular values of $\Delta t_{1,B}$, $\Delta t_{2,B}$, and $\Delta t_{3,R}$.

The game matrix is represented by the two functions:

$$\pi_{c,B}\left(\Delta t_{1,B},\Delta t_{2,B},\Delta t_{3,R}\right)$$

and $$\pi_{c,R}\left(\Delta t_{1,B},\Delta t_{2,B},\Delta t_{3,R}\right)$$

Blue picks $\Delta t^{*}_{1,B}$ and $\Delta t^{*}_{2,B}$ such that

$$\min\left\{\pi_{c,B}\left(\Delta t^{*}_{1,B},\Delta t^{*}_{2,B},\Delta t_{3,R}\right)\right\} \geq \min\left\{\pi_{c,B}\left(\Delta t_{1,B},\Delta t_{2,B},\Delta t_{3,R}\right)\right\}\Big|_{\Delta t_{1,B},\Delta t_{2,B}}$$

similarly Red chooses $\Delta t^{*}_{3,R}$ such that

$$\min\left\{\pi_{c,R}\left(\Delta t_{1,B},\Delta t_{2,B},\Delta t^{*}_{3,R}\right)\right\} \geq \min\left\{\pi_{c,R}\left(\Delta t_{1,B},\Delta t_{2,B},\Delta t_{3,R}\right)\right\}\Big|_{\Delta t_{3,R}}$$

The actual potentials are then

$$\pi_{c,B}\left(\Delta t^{*}_{1,B},\Delta t^{*}_{2,B},\Delta t^{*}_{3,R}\right) \quad \text{defined as} \quad \pi^{*}_{c,B}$$

and $$\pi_{c,R}\left(\Delta t^{*}_{1,B},\Delta t^{*}_{2,B},\Delta t^{*}_{3,R}\right) \quad \text{defined as} \quad \pi^{*}_{c,R}$$

Consider the Blue fire rate $v_{F,B}$ as a variable. At this point it is assumed that the quantity of ammunition required by Blue is simply $v_{F,B}\,\tau_{2,B}$ regardless of the particular course of action chosen. The supply manpower investment is $W\,\Delta t$ where $\Delta t = \tau_{2,B}$.

Then $\Delta W\,\Delta t \propto \Delta v_{F,B}\Delta t$

$$\Delta W\,\Delta t = C\;\Delta v_{F,B}\Delta t$$

where C = constant of proportionality in units of (man-days/quantity of ammo.)

The productivity μ is defined as

$$\mu = \frac{\pi_{c,B}}{W\,\Delta t} \qquad \text{and} \qquad \Delta\mu = \frac{\Delta\pi_{c,B}W\Delta t - \pi_{c,B}\Delta W\Delta t}{W\Delta t(W\Delta t + \Delta W\Delta t)}$$

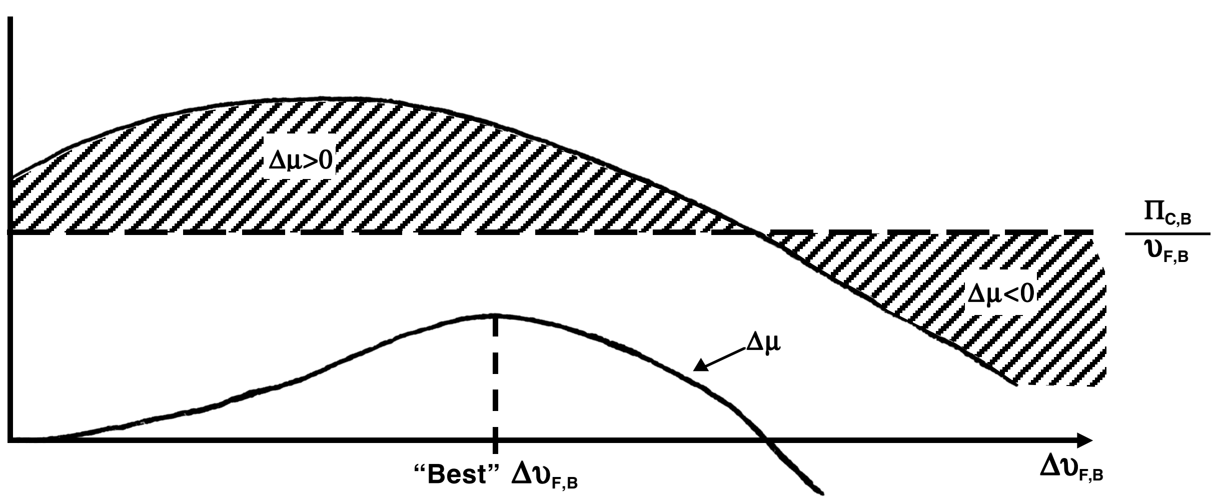
since we require that $\Delta\mu$ be greater than zero for an increase in productivity,

$$\frac{\Delta\pi_{c,B}}{\Delta W\Delta t} > \frac{\pi_{c,B}}{W\Delta t}$$

or

$$\frac{\Delta\pi_{c,B}}{\Delta v_{F,B}} > \frac{\pi_{c,B}}{v_{F,B}}$$

Solutions to this equation are graphically expressed by a curve of the following form.



Sketch B

The above curve identifies whether an increased fire rate is beneficial and if so how much of an increase should be designed to maximize tactical utility.

TACWAR #71-1
19 January 1973

RESULTS OF THE NUMERICAL EXAMPLE

DJGadler

Values for the constant parameters are chosen as follows:

D = 600 ft.

$\tau_{2,B}$ = 600 sec.

S = 15 ft/sec

$\sigma_B = \sigma_R =$ 1 milliradian

$v_{F,R}$ = 12 rounds/minute

$c_{\sigma,B} = c_{\sigma,R} =$ $1 + S/15$

$c_{V_{13}} = c_{V_{23}} = c_{V_{31}} = c_{V_{32}} = 0.125$

$A_{V,B} = A_{V,R} =$ 2 ft.^{2}

Q_I = 0.01 men

V_B = 1 man-second

V_R = 1 man-second

Values for the variable parameters are:

$\Delta t_{3,R}$ = 600 seconds

$\Delta t_{2,B}$ = 0 (no advantage exists for Blue to stop Δ_2 and return fire)

$0 < \Delta t_{1,B} < 180$ sec.

$6 < v_{F,B} < 36$ rounds per minute

Figure 1 shows $\pi_{C,B}$ as a function of $\Delta t_{1,B}$ parametric on $v_{F,B}$. Note that the optimum value of $\Delta t_{1,B}$ changes as expected for each $v_{F,B}$.

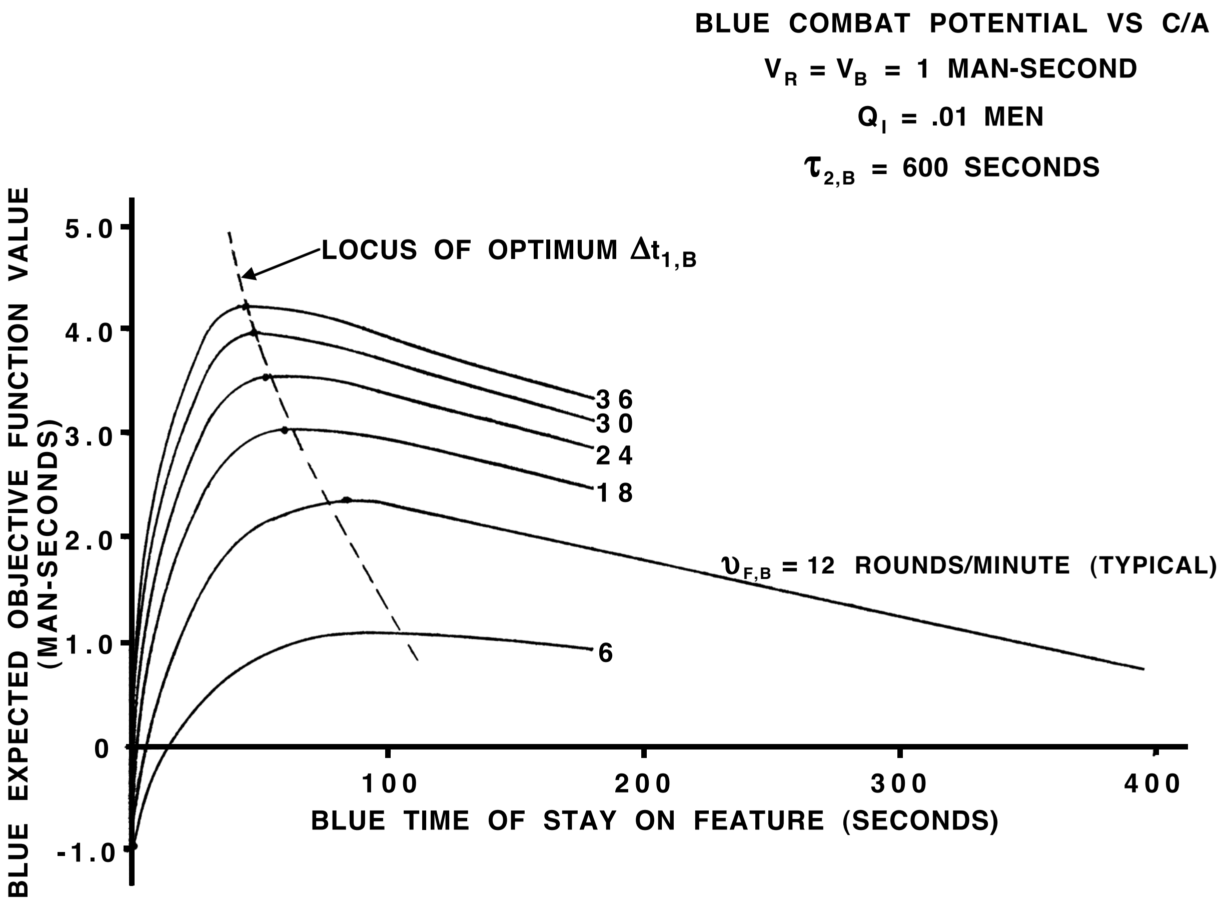


Figure 1

Figure 2 shows the optimum $\pi^*_{C,B}$ for each value of $v_{F,B}$.

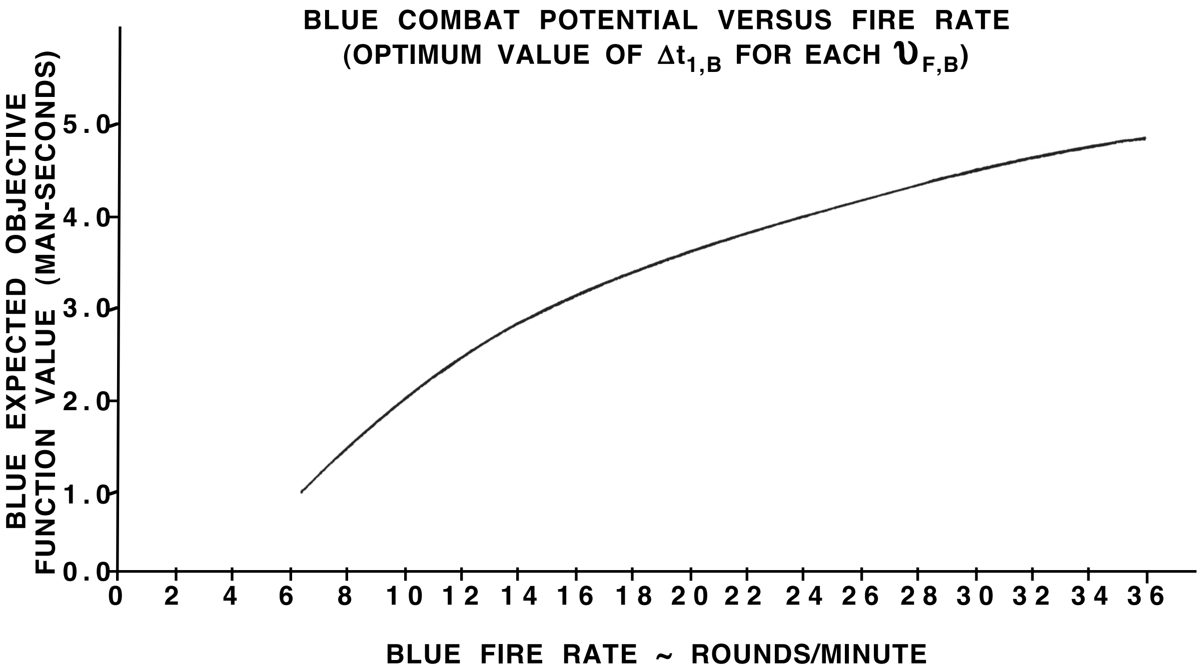


Figure 2

The equation for $\Delta\mu$ is

$$\Delta\mu = \frac{\Delta\pi_{c,B} W\Delta t - \Delta\pi_{c,B}\ \Delta W\Delta t}{W\Delta t(W\Delta t + \Delta W\Delta t)}$$

The manpower effort W is composed of two quantities, these being (1) W_o , which is the manpower effort to provide all necessary supplies other than ammunition, and (2) W_A which is the manpower required to provide ammunition. Assume that $W_A/W = 0.2$ and $W = 3$ men. Then,

$$W = C\ \Delta v_{F,B}$$

where, assuming $v_{F,B} = 0.2$ rounds/sec. Nominal

$$C = 3 \quad \text{man-sec./round}$$

For this example, Δt = 600 seconds so that $W\Delta t$ nominally equals 1800 man-seconds. The magnitude of the productivity is quite small, however the significance lies not in the numerical value of the results but rather the form of the results.

If $\Delta\mu$ is re-written as

$$\Delta\mu = \frac{\Delta\pi_{c,B} - \pi_{c,B}\ \Delta W/W}{(W + \Delta W)\ \Delta t} ,$$

We note that $1/\Delta t$ is simply a constant multiplier, therefore no significance is lost by normalizing the result and expressing rather than $\Delta\mu$, a $\Delta\hat{\mu}$ where

$$\Delta\hat{\mu} = \frac{\Delta\pi_{c,B} - \pi_{c,B}\ \Delta W/W}{W + \Delta W}$$

Both $\Delta v_{F,B}$ and $\Delta\pi_{c,B}$ are measured from the nominal values of 0.2 rounds/sec. and 2.35 man-seconds, respectively.

Plugging in the numbers:

$$\Delta\hat{\mu} = \frac{\Delta\pi\ \ - 2.35\ \Delta v_{F,B}}{3(1 + \Delta v_{F,B})}$$

The curve of $\Delta\hat{\mu}$ versus $\Delta v_{F,B}$ is shown in **Figure 3**. The fire rate which maximizes μ appears to be about 30 rounds/minute; however, any increase in $v_{F,B}$ up to some limiting value above 36 round/minute is desirable because of a corresponding increase in productivity.

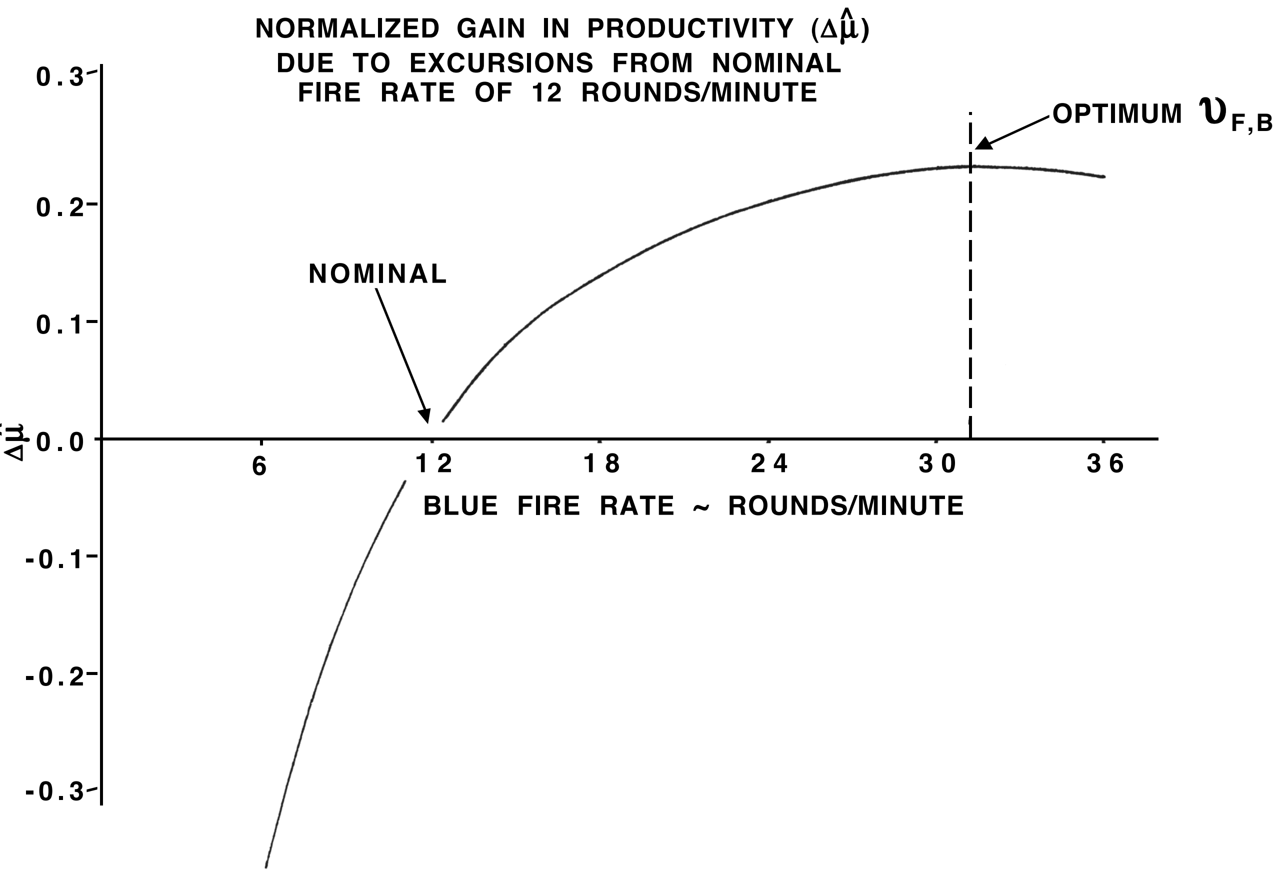


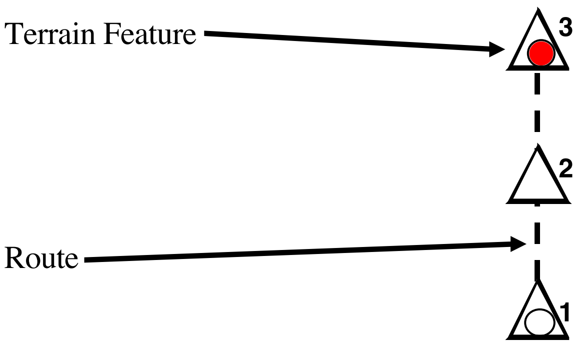
Figure 3

Following is a Computational Flow Diagram of the model described in TACWAR 71

Example

COMBAT INTERACTIONS - - - ELEMENTAL LEVEL

- Engagement Geometry:

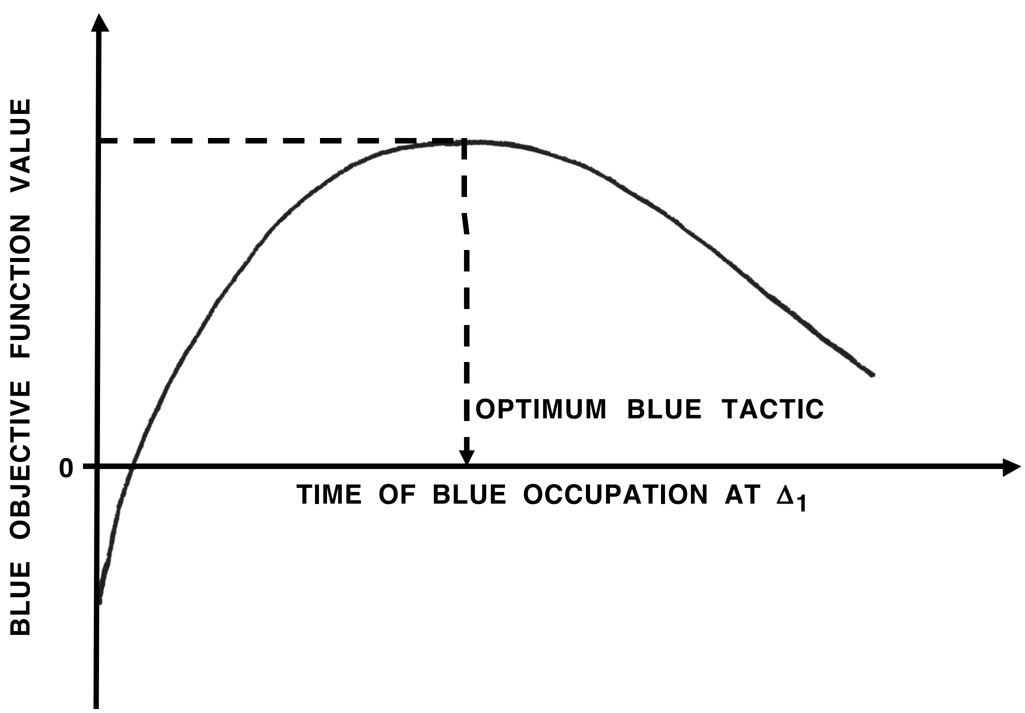


Blue's objective occupied by one Red unit.

Intermediate terrain feature

One Blue unit.

- Blue's Objective: Occupy feature Δ_3 before time t_2
- Tactics problem: How long should Blue return fire from Δ_1 before moving to take Δ_3?
- Solution:



COMPUTATIONAL FLOW DIAGRAM