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INTERDEPENDENCE OF COMMAND, FIRE, AND MANEUVER IN LAND COMBAT

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1. PURPOSE

The purpose is to develop a mathematical-logical model of the land combat situation schematically represented in Figure 1.

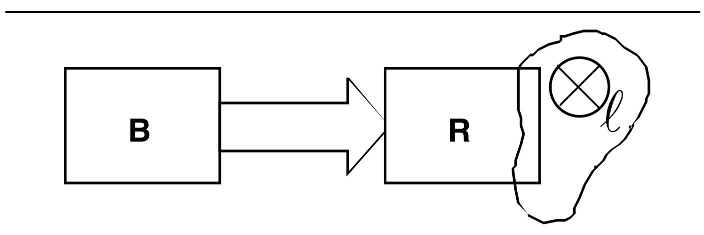

Figure 1

A Blue ground force is advancing, opposed by Red, for the purpose of capturing and occupying an objective area $\otimes_\ell$. The conflict is characterized by some rate of advance for Blue and by some level of casualties for both Red and Blue. The model must enable establishing these quantities, rate of advance and casualties, as a function of α , force mixes, η 's of Blue and Red, terrain characteristics and mission variables (Q, V_B, V_R) of Blue and Red.

2. SIMPLIFICATIONS

All nine military functions[1] are believed to influence combat performance. The key interface is believed to be among Command, Fire, and Maneuver; therefore, this document isolates these for analysis. The Intelligence function will make the problem considerably more complex, and is

[1] See *The Anatomy of Combat, 1996.* www.AnatomyOfCombat.com

deferred for later treatment. This requires that assumptions be made concerning the state, flow, and use of information; the following assumptions are used:

a) Each Player has the same knowledge of V and of the terrain as the Umpire (see Chapter 3, Reference 1). Neither player has any knowledge of the opponents Q, V_B, V_R .

b) Terrain characteristics of Concealment and Field of View are disregarded.

c) Players formulate no $O-[\ ]$ *, which rules out consideration of secrecy, deception and surprise.

The five logistic functions are considered only to the extent of incorporation of elemental stockage and weapon reliability in Φ[2].

Human factors are disregarded, except for some non-quantitative considerations of the possible effects of Skill and Motivation. For quantitative purposes both Skill and Motivation of all players are 1.00. In particular, Players can instantaneously estimate to some degree of precision as the Umpire. The Players do not exercise General ship.

3. PLAYER DECISIONS

Player decisions are based on a command matrix; the process is in three steps.

a) Planning establishes the dimensions of the Player's matrix and descriptions of the $O_I-[\ ]_I$ Since neither player knows the opposing Q, V_B, V_R , the Player matrices are generally large than the Umpire matrix. If the player matrices are $m \circ n$ (Blue) and $p \circ q$ (Red), the Umpire matrix is $m \circ q$.

b) Estimation evaluates the matrix entry for each $O-[\ ]$ according to the Player's own objective function:

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* NOTE: $O-[\ ]$ For clarity, O refers to Blue's Courses of Action. While $[\ ]$ refers to Red's Courses of Action .

2 See TACWAR 43.

For Blue:

$$P_{C,B}\Big|_{t_2-t_1} = \left(\left(t_2 - t_1\right) - \frac{d_{[\ ]_\ell,B}\Big|_{t_1}}{P_{M,B}\Big|_{t_2-t_1}} \right) Q^B_{I,[\ ]_\ell} + \sum_j \left(p_{R_j}\Big|_{t_2-t_1} \right) V^B_{R_j} - \sum_i \left(p_{B_i}\Big|_{t_2-t_1} \right) V^B_{B_i}$$

where

$$p_{R_j} = 1 - \exp\left[-\Phi_{O_F\, x\, R_j}\right]$$

and

$$p_{B_i} = 1 - \exp\left[-\Phi_{[\ ]_F\, x\, B_i}\right]$$

For Red, the function is similar, B's are substituted for R's, and vice versa.

Because of the assumptions regarding skill and generalship, matrix entry i , j in the three matrices has the following relations:

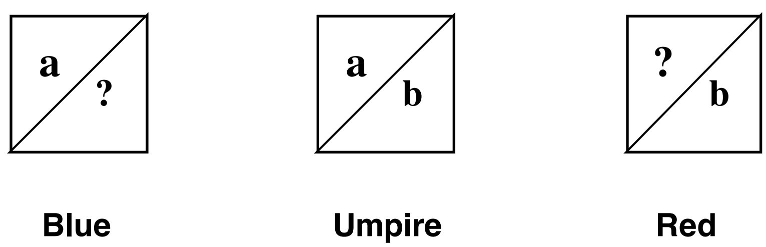

Figure 2

c) Decisions are made on a maximin principle. If at any time t , no favorable course of action exists ($P_{C,B}\Big|_{t_2-t_1} \leq 0$ for at least one matrix entry in each row for Blue), the Player reports that fact to higher command in the form of a Request (for change in orders). He then proceeds on the most favorable course of action to just short of a "point of no return". (See Paragraph 5)

Courses of action are reviewed, and the decision may be revised, after each critical event. Critical events are of two kinds: 1) Logic -Dependent (LD) which results from a decision by one Player, and 2) Probability-Dependent (PD) events, which are beyond the control of either Player.

The PD events in this simplified version of the problem are limited to "kills" [3] of elements, which are defined as removing all functional potential of the element involved until at least t_2. Before the event, kills are given a fractional "expected value" probability; when the event occurs, revision of plans is necessary. Contingency plans can be formulated in advance, based on assumptions as to time and place of kill. This is unnecessary for players who can instantaneously recalculate objective function values, as is assumed.

The LD events under the control of the opposing Player enter into the formulation of opponent's courses of action for matrix evaluation. Players defer final decision among own courses of action until a "point of critical decision" (see Paragraph 5) or the occurrence of an opponent-controlled LD event which enforces a single choice, whichever is earlier.

4. ANALYSIS OF A "ONE-ON-ONE" SITUATION

The situation is one in which two combat elements of equal K have a Q_I for the same terrain objective Δ_ℓ in the following Figure 3.

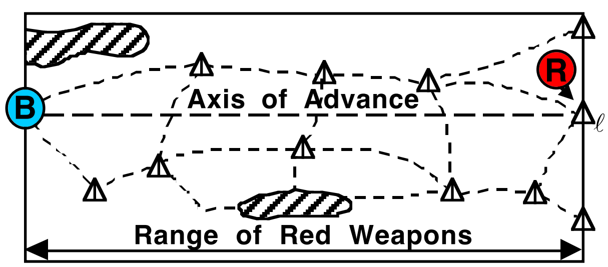

Figure 3

The elements will be described as if they were riflemen, but the same considerations can apply to other combat elements such as tanks, with rather obvious modifications -- such as that tanks ordinarily have little aptitude for changing posture. Both Blue and Red are limited in their movements by lateral boundaries, which are probably not the same for each. For initial simplification they are treated as the same, as are the $t_2 - t_1$ time periods for the two players. We analyze the problem only for the "battlefield" which includes a distance along the axis of advance equal to twice the maximum range of the weapons involved, which are assumed identical. Outside this distance movement is unopposed under the terms of this problem, since the opponent will be occupying Δ_ℓ with no incentive to leave.

[3] Weapon reliability failures have the same effect, and for simplicity are not separately discussed.

Blue has only three general courses of maneuver action available:

1. Move to Δ_ℓ , engaging in close combat if Red remains.

2. Move to some other Δ within the battlefield, from there attempting to kill Red or drive him out by fire.

3. Leave the battlefield because of no favorable course of action (make Request).

Red also has only three general courses of action available:

1. Remain at Δ_ℓ .

2. Move to some other Δ within the battlefield -- a "spoiling attack" or counter-attack by fire.

3. Leave the battlefield (making Request).

Each side has a continuous spectrum of courses of action for fire (O_F . $[\ ]_F$) because of control over τ_F .

4. OPPOSING COURSES OF ACTION

Under our assumptions each side can construct all possible $O-[\ ]$, and can estimate its own $P_C\big|_{t_2-t_1}$ at all times t according to its own Q, V_B, V_R . It cannot, however, estimate the worth to the opponent of any $O-[\ ]$ since it does not know the opponents' intentions Q, V_B, V_R .

Each O or $[\ ]$ should be thought of as a combination of a O_M and O_F (or $[\ ]_M$ of $[\ ]_F$) -- a Maneuver plan and a Fire plan. The maneuver plan is a schedule of stays on features (Δ) and movements on routes connecting features. The fire plan is a coordinated schedule for fire, specifying the target (only one possible in this example) and the proportional rate of τ_F at which ammunition is to be expended in each period. The course of action adopted by each opponent at t_1 will be that one which includes its maximin. These need not necessarily be at the same point for the two opponents because of the difference in objective function.

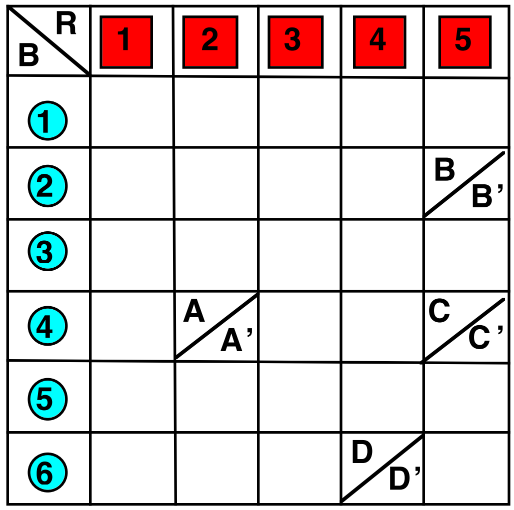

Figure 4

Thus in Figure 4 Blue may have chosen row O_4 because it contains his maximin A, while Red may have chosen column $[\ ]_5$ because it contains his maximin B'. If both persist the outcome will be C for Blue, C' for Red -- which by definition of maximin is better than expected for both sides.

However, when Blue sees Red committed to a course of action which precludes the column containing A, his expectation in row O_4 goes up -- his maximin is changed. He should not count on C since Red may still be able to enforce a different maximin -- say D, within the flexibility remaining to him. The same remarks apply to Red.

Every choice and execution of an LD event narrows the courses of action left open and may give the opponent an opportunity to improve his objective function expectation. The final $O-[\ ]$, that selected by the last LD event before t_2, was in the original matrix of possible $O-[\ ]$'s, but is not necessarily the intersection of the O and $[\ ]$ originally contemplated. This may verify the semi-serious military maxim that the winner is the one who makes the next-to-last mistake.

The LD decisions regarding maneuver are: Whether to remain at present Δ or if movement is contemplated, which Δ to move to. No decision is necessary regarding speed of movement. if an element has a capability for firing enroute, and speed affects K_F, the trade-off among Q_I, time spent in motion Δt_M, $p_B|_{\Delta t_M}$, V_B, $p_R|_{\Delta t_M}$, and V_R establishes the optimum speed.

Once a decision is made to stay, the stay is terminated only by a PD event, or by the initiation or successful completion of a move by the opponent. It could be terminated by a reduction of Φ due to enemy expenditure of ammunition; however, an opponent making perfect decisions will not expend ammunition in a fashion which makes a previously non-profitable move into a profitable one.

The danger in movement on a route is entirely dependent on the opponent's $\tau_F|_{\Delta t_M}$. He may fire at maximum rate for all of Δt_M, not fire at all, or do anything in between these two extremes. The risk (Φ) at any point will be a function of the range of that point from the enemy position. An element may move a short distance on a route, encounter heavy fire and return to position without accruing enough risk to make the attempt unprofitable. The point on the route at which the start and return is unprofitable is the "point of no return". If the point of no return is beyond the point of marginal risk, a point from which the enemy cannot impose sufficient Φ to make the whole move unprofitable, the attempt may be made. If heavy fire is experienced before reaching the point of marginal risk, the element turns back. However, for segments in which the point of marginal risk lies beyond the point of no return, the attempt is not made.

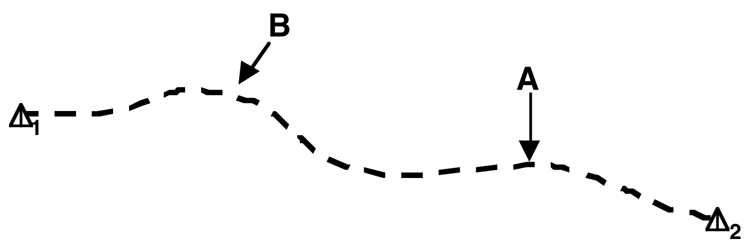

Figure 5

To be more specific, suppose that to move from Δ_1 to Δ_2 shortens the time-distance to the objective by $\frac{\Delta d_{\Delta_\ell}}{\eta_M K_M}$ (this may be less than Δt_M the time required for movement on the route if Δ_2 is off the most direct route to the objective). Then Blue can afford to take chances up to $Q_I^B \frac{\Delta d_{\Delta_\ell}}{\eta_{M,B} K_{M,B}}$ in order to get to Δ_2. If V_B^B times the chance taken to Point A and back is equal to $Q_I^B \frac{\Delta d_{\Delta_\ell}}{\eta_{M,B} K_{M,B}}$, then A is the point of no return. If V_B^B times the chance taken from point B on is equal to $Q_I^B \frac{\Delta d_{\Delta_\ell}}{\eta_{M,B} K_{M,B}}$, then B is the point of marginal risk. If Blue makes the attempt and Red initiates fire beyond point B, then Blue is ahead. If Red initiates fire before point B, Blue can return to Δ_1 without having incurred an unacceptable risk. In effect he gambled for $Q_I^B \frac{\Delta d_{\Delta_\ell}}{\eta_{M,B} K_{M,B}}$ and lost, but the gamble was necessary since he did not know Red's $[\;]_M$ -- Red might have run.

Every shot fired is also an LD event. The $[\ ]_F$ (and inversely the O_F) is constructed originally on the $O-[\ ]$ which is a maximin from the Red point of view. Knowing $[\ ]_M$ and assuming O_M , he can plan to utilize his available fire ($\int \tau v_F \, d\tau$ = elemental stockage) during intervals of maximum Φ for Blue.

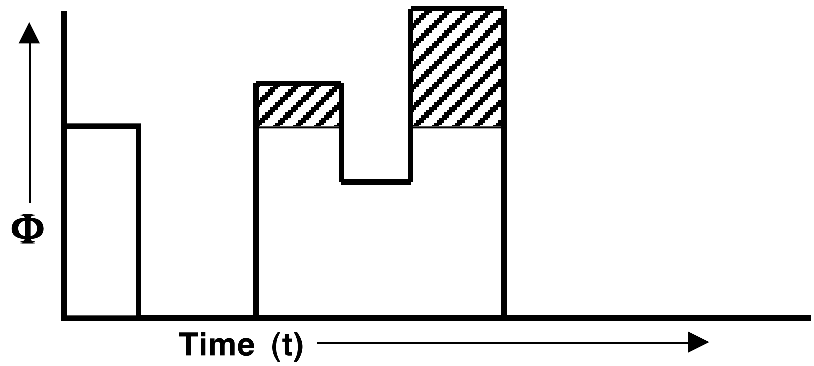

Figure 6

If Blue's assumed pattern of exposure is as shown above, he may distribute his available ammunition to the shaded areas; in effect he is setting a minimum ΦV_B at which he will fire. If Blue makes a move which rules out the previous Red maximin, the fire plan can be re-done and a lower minimum ΦV_B can be set, utilizing the ammunition not yet expended.

If Red estimates an ability to remain on the objective, he will be well-advised to reserve his fire for use against Blue on <u>routes</u> at minimum range (Provided Blue cannot mount suppressive fire which lowers Red's η_F). On the other hand if he plans to leave Δ_F at some point he may expend his ammunition before departure, and thus at greater ranges.

In the illustrative one-on-one problem Blue must make the first logical decisions. He must assign three general $[\ ]_M$ -- 1) to leave the battlefield, 2) to remain at Δ_ℓ , or 3) to move to some other Δ within the battlefield. Since Red is in defense and has no Q_I^R for any point other than Δ_ℓ , it is very probable that Δ_ℓ is the strongest defensive point in the battlefield -- the one which makes $\Phi_{[\ ]_F\ xB} - \Phi_{OF\ xB}$ greatest for any O_M. Blue must still evaluate possible threatening moves by Red, but he is likely to find his maximin in some $O_M - [\ ]_M$ which involves Red's remaining at Δ_ℓ .

Blue has two generalized courses of action: 1) In Assault, he moves directly to Δ_ℓ and, if Red has not retreated, engages him in close combat; 2) In Attrition, he moves to his most favorable firing point Δ_F and attempts to kill Red by fire, subsequently moving unopposed to Δ_ℓ .

Since he can at least come to Red's extreme range, Blue's first logical decision is a selection of entry point to the battlefield. To do this involves selection of a route for Assault and a route for Attrition; a process which can be described in terms of "backward planning"[4].

An Assault route, unless Red's time of departure from $\Delta_\ell\left(t_{D,R}\right)$ precedes Blue's time of arrival $\left(t_{A,B}\right)$, or unless some kill or reliability failure occurs before $t_{A,B}$, must end in Close Combat. Close combat is an engagement with secondary weapons or at least with primary weapons used in a non-standard way (clubbed rifle or shooting from the hip). Under our assumptions of infallible logical decision, Close Combat is either: 1) Impossible, 2) Final, or 3) Avoided by one side at least.

Close Combat is impossible for certain pairs of opposing elements, e.g., tanks. They can do no good by ramming each other, or with secondary armament, and therefore will elect to maintain some minimum range. This will be dictated by weapon traverse and aiming difficulties at too short a range.

If Close Combat occurs it is because both sides have advanced to a common point, or because one has advanced and the other has not retreated. Retreat is characteristically more dangerous than either advance or maintenance of position. An element making logical errorless decisions, having accepted a certain risk to gain the probability of victory in close combat, will not undergo a greater risk to avoid the probability of defeat in close combat. If the odds are bad according to objective function, the element will take all necessary measures to avoid close combat.

Blue accepts Close Combat (if he gets that far) provided

$$Q^B_{I,\Delta_\ell}\left(t_2 - t_{A,B}\right) \geq p_{B(CC)}V^B_B - p_{R(CC)}V^B_R$$

Regardless of the answer, Blue continues to evaluate an Assault O_M. Red may retreat before Blue reaches the point of critical decision at which he logically must abandon an Assault O_M. He next evaluates his advance from an Assault position -- a chosen feature directly connected to Δ_ℓ by a route having no intermediate Δ. The chosen assault position will usually be that one closest in terms of movement time $\left(\Delta t_{M_B}\right)$ to Δ_ℓ. Blue evaluates the advance from Assault Position by :

[4] Backward planning as herein described may sometimes result in selection of a less-than-optimum route; therefore, we retain the picture that Blue considers all available routes -- we are mere describing the process from the forward end of the routes.

$$Q^B_{I,\Delta_\ell}\left(t_2 - t_{A,B}\right) \geq \left\{1 - \left(1 - p_B\big|_{\Delta t_M}\right)\left(1 - p_{B(CC)}\right)\right\} V^B_B - \left\{1 - \left(1 - p_R\big|_{\Delta t_M}\right)\left(1 - p_{R(CC)}\right)\right\} V^B_R$$

If yes, the Assault O_M is still favorable. If not, Blue can calculate a point of no return and point of marginal risk to determine whether the attempt should be made. if the attempt is not profitable, Assault by this route has a negative minimum value, assuming that Red maintains position. However, Blue must continue to determine the most profitable Assault route, even though the objective function expectation is negative. The process is simply an iteration by successively rearward route segment of that described above.

Because of the rapidly-decreasing range and increasing $\Phi_{[\ ]_F\ x\ B}$ on the advance from the assault position, it is unlikely that that move can be favorable unless Blue can use Assault Fire. Assault Fire is a form of suppressive fire in which Blue, firing at maximum rate and advancing steadily, makes it wiser for Red to keep his head down and leave the issue to close combat. In effect both $\eta_{F,B}$ and $\eta_{F,R}$ go to 0 because $\eta_{F,B} \cong 1.00$. Red decides to keep his head down if:

$$\left\{1 - \left(1 - p_R\big|_{\Delta t_{AF}}\right)\left(1 - p_{R(CC)}\right)\right\} V^R_R - \left\{1 - \left(1 - p_B\big|_{\Delta t_{AF}}\right)\left(1 - p_{B(CC)}\right)\right\} V^R_B \ \geq\ p_{R(CC)} V^R_R - p_{R(CC)} V^R_B$$

where Δt_{AF} = period of Assault Fire.

Having determined the most profitable Assault O_M , Blue proceeds to determine the most profitable Attrition O_M. He first selects that feature (Δ_F) which offers the best duel potential. He realizes that both $\Phi_{O_F\ x\ R}$ and $\Phi_{[\ ]_F\ x\ B}$ can be applied at maximum rate until one opponent is killed -- or until Red fires down to the safety level of ammunition he may desire to hold to preclude an Assault by Blue. Blue cannot estimate the latter, however, since Red may be merely trying for a kill and planning a retreat if unsuccessful. If $\Phi_{[\ ]_F\ x\ B} > \Phi_{O_F\ x\ R}$ because Red occupies the best position in the area, the expectation is that Blue will be killed <u>before</u> Red, but there is a chance that the reverse will be true. it is necessary to calculate the expected time of Red kill ($t_{K,R}$), given that Blue is not killed.

$$p_R\big|_{t_2 - t_{K,R}} = p_R\big|_{t_{K,R} - t_1}$$

Blue considers the attrition O_M favorable, given that he reaches Δ_F , if

$$\left\{Q^B_{I,\Delta_\ell}\left(t_2 - t_{K,R} - \frac{d_{\Delta\ell}\big|_{t_{\Delta F}}}{\eta_{M,B}K_{M,B}\big|_{t_{\Delta F}}}\right) + V^B_R\right\}\left(p_R\big|_{t_2 - t_{\Delta F}}\right) \geq \left(p_B\big|_{t_2 - t_{\Delta F}}\right)V^B_B$$

$t_{\Delta F}$ being Blue's time of arrival at Δ_F .

If this is favorable, Blue retains an Attrition O_M as an option, and proceeds to calculate his best route to Δ_F , and the probability of reaching it, as for the Assault O_M. He need, of course, calculate only back to a point at which this route merges with the best assault route. This is a point of critical decision for Blue -- when he reaches it he must make his final choice between Attrition and Assault by a maximin reading. As previously noted, if he goes to an Attrition mode, Red will have no reason to retreat if he has not already done so.

Red, being on the defense, usually has all the advantages in this one-on-one encounter between equals, not only in chance of survival, but in simplicity of estimation. He has only two principal $[\ ]_M$ -- to stay at Δ_ℓ, or to retreat.[5] He retreats when ($t_{D,R}$ = time of departure).

$$Q^R_{I,\Delta_\ell}\left(t_2 - t_{D,R}\right) \leq \left(p_R\big|_{t_2 - t_{D,R}}\right) V^R_R - \left(p_B\big|_{t_2 - t_{D,R}}\right) V^R_B$$

He avoids being trapped at Δ_ℓ by calculating the p_B and p_R for Red stay and Red retreat. A $t_{D,R}$ necessary to avoid being trapped is a time of critical decision. This will characteristically occur during a forward move by Blue; if the equality occurs with Blue at a feature Red will initiate his move just as Blue leaves that feature on a route which will make Red's expectation negative.

After Red leaves Δ_ℓ he can have no incentive to return or to halt at any other Δ, because his expectation is clearly negative until he reaches a point where $\left(p_R\big|_{t_2 - t}\right) V^R_R \leq \left(p_B\big|_{t_2 - t}\right) V^R_B$. Obviously, since he has retreated from a good defensive position, $V^R_R > V^R_B$.

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5 He might leave Δ_ℓ for a short period to take advantage of an unusual opportunity to increase $\Phi_{[\ ]_F\, x\, B}$, but with our assumptions Blue will not offer such a chance.

6. OUTCOMES OF THE ONE-ON-ONE SITUATION

Each opponent can, and will, adjust his O_M (or $[\ ]_M$) to limit his exposure to fire so that his objective function expectation is positive. This provides a convenient classification for outcomes, according to whether neither, either, or both must limit their O_M (or $[\ ]_M$) .

At this point the umpire has all the information needed to establish the probabilities of various outcomes. He can establish $\tau_{M,B}$ as a function of the $Q's$, $V's$, $K's$, ϑs and terrain. Neither player has this ability, however, although the only element lacking is a knowledge of the enemy's intentions -- his Q_I , V_B , V_R . Unless he is practicing Generalship each player must include one adverse course of action that is most pessimistic for him. Red assumes that Blue may assault; Blue that Red may maintain position.

The Umpire has four different methods of solution, classified as shown.

○M \ □M	Unlimited	Limited
Unlimited	**A**	**B**
Limited	**C**	**D**

Figure 7

Referring to Figure 7:

A. Neither opponent limits exposure to fire. In this case, both opponents take that course of action which maximizes objective time. Blue proceeds directly to Δ_ℓ with $t_{A,B} = \dfrac{d_{\Delta_\ell}\big|_{t_1}}{\eta_{M,B} K_{M,B}\big|_{t_1}}$. Red remains at Δ_ℓ , and the conflict is finally resolved by close combat unless one opponent has been previously killed.

$$Q^B_{I,\Delta_\ell}\left(t_2 - t_{A,B}\right) > \left(p_B\big|_{t_2 - t_1}\right) V^B_B - \left(p_R\big|_{t_2 - t_1}\right) V^B_R$$
$$Q^R_{I,\Delta_\ell}\left(t_2 - t_1\right) > \left(p_R\big|_{t_2 - t_1}\right) V^R_R - \left(p_B\big|_{t_2 - t_1}\right) V^R_B$$
$$p_B\big|_{t_2 - t_1} = 1 - \left\{\left(1 - p_B\big|_{t_{A,B} - t_1}\right)\left(1 - p_{B(CC)}\right)\right\}$$

$$p_R\big|_{t_2-t_1} = 1-\left\{\left(1-p_R\big|_{t_{A,B}-t_1}\right)\left(1-p_{R(CC)}\right)\right\}$$

$$P_B\big|_{t_2-t_1} + P_R\big|_{t_2-t_1} = 1.00$$

If Blue is killed $\tau_{M,B} = 0$

If Red is killed $\tau_{M,B} = 1.00$

$\eta_{M,B}$ depends on trafficability of routes. $d_{\Delta_\ell}\big|_{t_1}$ depends on the dimensions of the battlefield, $p_B\big|_{t_{A,B}-t_1}$ depends on Red's K_F against a moving target. $p_R\big|_{t_{A,B}-t_1}$ depends on Blue's K_F and cover ζ at Δ_ℓ

B. Red limits exposure to fire, but Blue does not. In this case Blue again assaults by the most direct Route, and Red retreats before close combat is joined.

$$Q^B_{I,\Delta_\ell}\left(t_2-t_{A,B}\right) > \left(p_B\big|_{t_2-t_1}\right) V^B_B - \left(p_R\big|_{t_2-t_1}\right) V^B_R$$
$$Q^R_{I,\Delta_\ell}\left(t_{D,R}-t_1\right) > \left(p_R\big|_{t_2-t_1}\right) V^R_R - \left(p_B\big|_{t_2-t_1}\right) V^R_B$$

$$t_{D,R} - t_{A.B} > 0$$

If Blue is killed $\tau_{M,B} = 0$

Otherwise $\tau_{M,B} = 1.00$

Red may be leaving simply to avoid the finality of close combat. In this case he does not leave the battlefield but takes up a Δ_F in an Attrition mode. This affects both $p_B\big|_{t_2-t_{D,R}}$ and $p_R\big|_{t_2-t_{D,R}}$.

C. Blue limits exposure to fire, but Red does not. In this case Blue goes into an Attrition phase on some Δ_F , while Red remains at Δ_ℓ .

$$\left(p_R\big|_{t_2-t_1}\right) Q^B_{I,\Delta_\ell}\left(t_2 - t_{K,R} - \frac{d\big|_{\Delta_F}}{\eta_{M,B} K_{M,B}\big|_{t_{K,R}}}\right) =$$
$$\left(p_B\big|_{t_2-t_1}\right) V^B_B - \left(p_R\big|_{t_2-t_1}\right) V^B_R$$

$$Q^R_{I,\Delta_\ell}\left(t_2 - t_1\right) > \left(p_R\big|_{t_2 - t_1}\right) V^R_R - \left(p_B\big|_{t_2 - t_1}\right) V^R_B$$

If Blue is killed $\quad \tau_{M,B} = 0$

If Red is killed $\quad \tau_{M,B} = \dfrac{d\big|_{t_1} \Big/ \eta_{M,B} K_{M,B}\big|_{t_1}}{t_{K,R} + \dfrac{d\big|_{t_{K,R}}}{\eta_{M,B} K_{M,B}\big|_{t_{K,R}}} - t_1}$

If neither is killed, $\tau_{M,B} = \dfrac{d\big|_{t_2}}{d\big|_{t_1}}$

D. Both sides limit exposure to fire.

This case is a game of "chicken"; it can end in two ways (other than kill of one opponent).

Blue guides his actions by :

$$Q^B_{I,\Delta_\ell}\left(t_2 - t_{A,B}\right) = \left(p_B\big|_{t_2 - t_1}\right) V^B_B - \left(p_R\big|_{t_2 - t_1}\right) V^B_R$$

However, he progresses toward Δ_ℓ only to a critical time t_C at which he must make a final choice between Assault and Attrition. Red guides his actions by:

$$Q^R_{I,\Delta_\ell}\left(t_{D,R} - t_1\right) = \left(p_R\big|_{t_2 - t_1}\right) V^R_R - \left(p_B\big|_{t_2 - t_1}\right) V^R_B$$

If $t_D > t_C$, Blue goes into an Attrition mode

If $t_C > t_D$. Red Retreats