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NOTES ON THE ACCURACY OF PROJECTILES

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1. This paper is intended to further the analysis of the Fire Function by systematizing the approach to weapon system accuracy. Accuracy is very determinative of fire element capability (as distinguished from potential [1]). In fact, accuracy, lethality, and reliability combine to determine capability; all other factors being situation-dependent are relegated to the more difficult determination of potential. Lethality has been partially discussed [2]; more work is needed. In Paragraph 3, below, the relationship of reliability to accuracy is discussed.

2. The capability of a Fire Element can be expressed as

$$R \int \Pr\left[\upsilon_D\right] \ L(\upsilon_D) \ d\upsilon$$

where R is reliability; υ identifies points in a multiple dimension vector space describing the projectile state (location, orientation, velocity) and υ_D is υ at detonation. L describes lethality as a function of υ_D . $\Pr[\upsilon_D]$ is a probability density function (p.d.f.) for detonation at υ_D . The indicated integration is performed over the domain of υ for which $L \neq 0$. $L(\upsilon_D)$ is determined by analysis of post-detonation events; $\Pr[\upsilon_D]$ measures accuracy and is determined by analysis of pre-detonation events. Thus, the event detonation separates, for analysis purposes, the concepts of accuracy and lethality.

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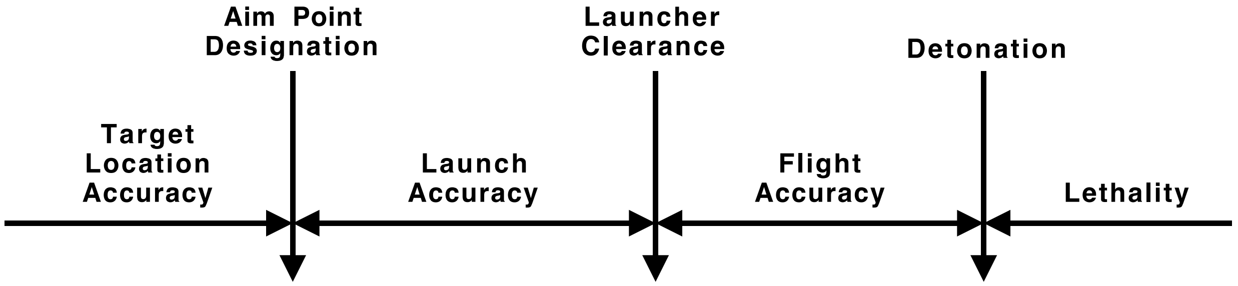
[1] See Chapter 3.0 Functional Representation of Combat; *THE ANATOMY OF COMBAT*, or ---www.The Anatomy of Combat.com on the Internet.

[2] See TACWAR 7. Target Modeling and Weapon Capability Analysis, dtd 15 Mar 71.

3. Similarly, the general concept of accuracy is divided into three parts by two other events as follows:



Launcher clearance denotes the termination of any mechanical interaction of launcher and projectile. The aim point, υ_D^* , is designated as a first step in the computation of a desired trajectory; its location (and error) is the result of processes of Intelligence (Target Acquisition) and Command (Tactical and Technical Fire Direction).

The target location error (t.l.e.) is clearly independent of subsequent errors. It does contribute, often very significantly, to the difference between optimum υ_D and actual υ_D. (In two cases, direct fire or homing weapons, the t.l.e. is insignificant.) Whether t.l.e. is included in "weapon system error" is a matter of taste - it depends on how "weapon system" is defined. In many cases, its inclusion is merely confusing. The intelligence and command networks and procedures of the force should be studied separately. In any case, target location accuracy is not a subject of this paper. See [3] for some discussion thereof. Following the approach of [4], target location accuracy would be analyzed by study of the Intelligence function.

4. Launch Accuracy and Flight Accuracy, plus another category to be introduced later (Guidance Accuracy) are the subject of this paper; however, before proceeding to the discussion, it is pertinent to establish a conceptual approach to two other factors - Reliability and Countermeasures.

Accuracy is dependent in a very complex way on the proper functioning of all weapon components during the interval between launch initiation and detonation - in other words, on a certain segment of overall weapon system reliability. Complete failure of any weapon component during that interval will probably lead to very large miss distances which cannot be included in the probability density function (p.d.f.) for accuracy without large distortions thereof. On the other hand, even partial failures will have an effect on accuracy and often in a fashion which produces a bias error rather than a symmetric distribution of error. As an example, some relatively small defect in a propulsion

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[3] Ibid.
[4] Ibid.

system may produce a deficiency in thrust and this will weight the distribution of round toward "shorts" rather than "longs". For precision of understanding of the analysis, it is important to define a reliable weapon as one in which all the various components operate, at appropriate times on receipt of appropriate specified inputs, within certain defined and general symmetric ranges of functional performance. Reliability is then a measure of the probability density function of υ_D , given a reliability of 1.00.

Countermeasures, being situation-dependent, enter the analysis of potential rather than capability. They are discussed at this point merely because of their analytic relationship to Reliability. Some countermeasures (hardening and dispersal of targets) affect weapon lethality rather than accuracy. Those countermeasures which affect accuracy can do so in two modes: 1) By changing the characteristics of the target or target scene (camouflage, decoys) in such a way that erroneous observations are made by components of the weapon system; or 2) By directly interfering with (jamming) the functioning of weapon components. The first mode affects certain guidance probabilities to be discussed later (Para. 12). The second mode should be viewed as affecting Reliability. If it becomes important to assess accuracy under countermeasures, more than one standard of Reliability (as affected by countermeasures) can be defined for accuracy investigations.

5. The concept of Accuracy having been isolated from those of Reliability and Lethality, it is now possible to describe and classify sources of error to indicate the necessary algorithms for their evaluation and to comment on required inputs and their possible availability in empirical data. The attempt is to identify independent sources of error, or at least sources with such weak correlation that they can be treated as independent without serious inaccuracy. Full generality is sought in identification of all components of the vector $\upsilon_D - \upsilon_D^*$. In particular, the sometimes useful and sometimes misleading concept of circular probable error ("c.e.p.") is avoided in favor of a tri-variant p.d.f. for location at detonation. The analysis is applicable to all projectile weapons discharged from any mechanical launcher, including zero length. (The human arm could be considered a "launcher" but is excluded from consideration herein.)

6. Projectile weapons are classified herein in three different fashions: by aiming conditions, by guidance methods, and by fuzing methods.

6.1. Aiming Conditions:

6.1.1. Direct Fire weapons are those in which the target is observable from the launcher and is used as a reference for aiming.

6.1.2. Indirect Fire weapons are those in which the target is not used as a reference in aiming whether or not it is observable from the launcher.

6.2. Guidance Methods:

6.2.1. Unguided ballistic projectiles are those in which no post-launch propulsion or control forces are applied.

6.2.2. Programmed projectiles are those in which post-launch propulsion or control forces, or both, are applied according to a time schedule established pre-launch and not dependent on any observations made while the projectile is in flight.

6.2.3. Corrected projectiles (midcourse guidance) are those in which post-launch control forces are applied based upon observations of differences between the actual trajectory and the planned trajectory. However, no observations are made of the target scene containing an aim point.

6.2.4. Guided projectiles (terminal guidance) are those in which post-launch control forces are applied based upon observations of the aim point in a target scene; these observations are made by the projectile itself or some other element, termed a controller.

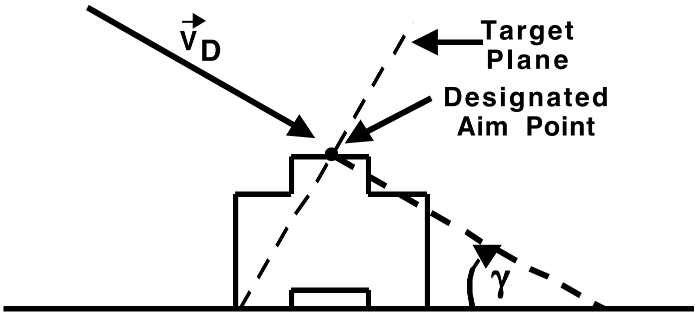
6.2.5. Homing projectiles are those in which post-launch control forces are applied based upon observations of the target itself. In most cases, the observations are made by the projectile itself, but the case where they are made by a controller is not excluded. A special case of homing projectiles is that in which illumination of the target by an illuminator is required.

Any one projectile may combine two or more of the above modes into a single trajectory. in cases where the projectile passes to a corrected, guided or homing phase, the demarcation from the previous phase is determined by an event called "First Correction" (C1). In such case, analysis of error in preceding phases is designed to give a p.d.f. for υ_{C1} (state at first correction. Analysis of error (guidance accuracy) in subsequent phases establishes a p.d.f. of υ_D as a function of υ_{C1} . One useful approximation to this notion is to speak of a "window" or "basket" which must be attained to enable successful correction, guidance, or homing. Of course, in these cases the Guidance Accuracy (this term is used for "corrected" or "homing" projectile or phase also) after event C1 is likely to be dominant over accuracy in the earlier phases.

6.3. Fuzing Methods:

Fuzing determines the point of detonation, and is of four general kinds: Command, Time, Influence, and None. Analysis of the fuzing method establishes which of the components of υ_D are of principal interest to the analysis of accuracy.

A projectile having no fuzing, of course, has no actual detonation. The υ_D is retained in symbology, but actually refers to projectile impact. The projectile is then of a type which achieves its effect by impact on the target element. If the lethality analysis is of great detail, the vector $\vec{v}$ and the orientation angles α, β, and ϕ may be of interest, together with a detailed description of the target surface seen in direction $\vec{i}_{v_D}$ and referred to a set of target (T) axes. However, normally a suitable approximation is to incorporate these considerations into an average omni-directional L ; in such case, accuracy is determined by y_T and z_T on passage of the target plane. If these values fall outside a projection of the target on a target plane normal to $\vec{v}_D$, L becomes zero; otherwise, L has its average value. The termination of the trajectory can be considered fixed by the x_T of the target's center. This will usually be the designated aim point and thus $x_T \cong 0$ at impact.



Influence fuzes are those that function based on some observation of the current environment of the projectile. The observation may be directly of the environment, as in VT fuzes; or it may be of the influence of the environment on the projectile, as in contact fuzes. Regardless of the method and technical details of the fuze, it is always possible to define a "fuzing surface" which is an envelope of all points in the target scene at which a fuze of standard performance characteristics will operate. If orientation is important to fuze operation, the fuzing surface must be define in υ-space. The size of target scene to be modeled (and, incidentally, the limits of integration over υ) are determined by the lethal radius, r_L · of the warhead and the radius of the target r_T , because it is of little use to study fuzing at points where a detonation will not affect the target.

The modeling of the target scene may, of course, be as detailed as desired; but two general approximations are of principal interest. For fuzes which operate on a height of burst principle, the fuzing surface is a horizontal plane with z_e = h.o.b. (for contact fuzes, h.o.b.= 0). For fuzes which operate on the observation of some pre-set distance from any object, the fuzing surface is the locus of all points at that distance from objects in the target scene which will activate the fuze. (Those "objects" may, of course, include the ground or water surface, dependent on fuze response characteristics.) In cases where target orientation is random or unknown, the fuzing surface is approximated by some figure of revolution about a vertical axis, the figure being embedded on a horizontal plane at fuzing distance from the ground.

Fuzes operating on a height-of-burst principle are often used to maximize the effect of some fragmenting warhead against target elements dispersed around some aim point. The assumption of a horizontal fuzing plane which is made for accuracy calculations shouldn't be extended to lethality studies. The detailed relief of the target scene, and the positioning of target elements therein, has a significant effect on the lethality, L.

The operation of influence fuzes is independent of the previous history of the projectile, except in cases where that history leads to some fuze malfunction which is comprehended under Reliability. Therefore, for Accuracy calculations, fuze errors are independent and may be vectorially combined with the errors arising from other sources. In most cases the fuzing performance and fuzing error override all other errors in one or more of the dimensions of υ_D . For instance, for height-of-burst fuze $z_e(\upsilon_D)$ is a constant ± the fuzing error. This knowledge can lead rather directly to a calculation of error in time of flight, t_D (assuming time = 0 at launcher clearance).

Time fuzes operate at the end of some pre-set time interval beyond some previous event in the trajectory. Their performance and error thus directly determine the time of flight and its error.

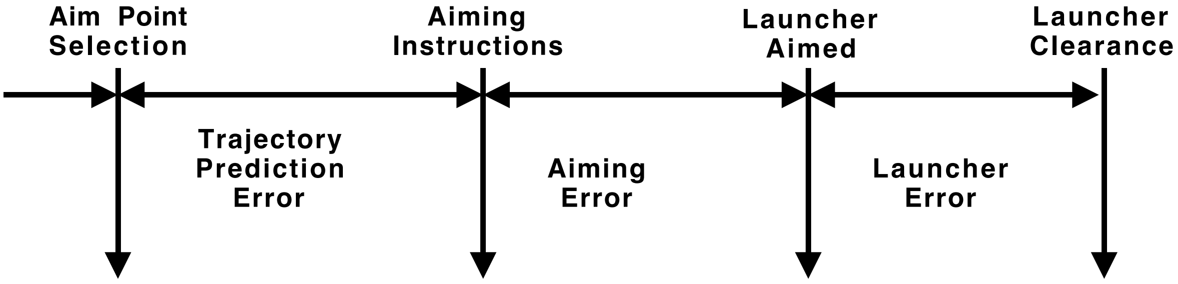
The probable error of time fuzes should be eminently easy to determine; if necessary, the entire production can be assessed for performance under conditions simulating those of actual flight. In fact, if desirable the fuzes can be calibrated and lotted. If the time starts operating at launch, the timer error determines the error in time of flight. If its operation commences at some other event, the error in time of flight has another component - the error in time interval between launch and that other event.

Similarly, the probable error of the influence fuzes should be easy to determine. It can be attributed to two independent components - error in reading of the pertinent "influence" and error in the predicted time of response between observation and detonation.

Command fuzing introduces errors very similar to those of command guidance; therefore, discussion of this method of fuzing is deferred to the discussion of command guidance.

7. Launch error is measured by $\vec{v}_L - \vec{v}_L^*$, the difference between the actual velocity at launcher clearance and that launch velocity which, under the existing environmental conditions, would have brought a standard projectile exactly to the designated aim point υ_D. $\vec{v}_L - \vec{v}_L^*$ is an unpredicted component of velocity, and for a standard projectile it will produce an error $\left(\vec{v}_L - \vec{v}_L^*\right) t_D$. This error is not truly independent of subsequent flight errors engendered by any non-standard characteristics of the projectile. This is because the reactions of the non-standard missile will cause it to fly through a wind regime somewhat different than that which would have been experienced by a standard missile. It may be defensible, however, to assume interdependence of launch error and flight error on the basis that the small trajectory deviations are trivial on the rather large scale of wind phenomena.

The launch error can be subdivided as indicated in the following diagram:



It is important to realize that a trajectory is always predicted. Even when a rifle is simply aligned with the designated aim point, with no allowance for elevation or windage, the prediction is of a straight-line trajectory. It is a poor prediction, but a prediction nevertheless.

The operation normally performed is to calculate a trajectory which will take a standard ("zero-tolerance) projectile from a launcher to υ_D^* in the predicted environment (environment implies pressure, true wind, temperature, and any other natural factor which may influence the flight of a standard projectile). The calculation for a standard missile in any precisely-described environment can be made, using ordinary laws of aerodynamics, to any precision desired. As a practical matter, it is useless to seek a degree of precision beyond that which makes computational errors (approximations) minor in comparison to other system errors. The trajectory prediction error, excluding computation error, has two components - launcher location error and wind force prediction error.

υ_D^* is fixed in some reference system; in the notation adopted for this analysis, it is the origin of the earth-axes (e). The computation of planned trajectory must start with the establishing of a vector for launcher position in terms of e-coordinates - what is often referred to as a gun-target line. The errors in this procedure are entirely geodetic, and this is the only geodetic process involved in projectile accuracy (as distinguished from target location accuracy) so that the launcher location error is an independent component of the trajectory prediction error. Launcher location error can be largely reduced by precision geodetic methods; but, as for the computational errors discussed above, it is wasted effort to strive for precision beyond that which makes these errors minor in comparison to other system errors. For moving launchers, such as airborne or shipborne, th error in prediction of location at firing time is the launcher location error.

The wind force prediction is based upon the performance of a standard projectile flying through a predicted environment on a trajectory which terminates at υ_D^* . The predicted environment establishes the values of ρ, the density of the air, and $\vec{v}_W$, the true wind velocity, at all points of interest. The standard projectile is considered to have exactly nominal characteristics (weight, shape, stability, smoothness, etc.). With these precise inputs, the aerodynamic problem is solved to yield a required velocity at launch $\left(\vec{v}_L^*\right)$. Since speed, $\left| \vec{v}_L \right|$, is fixed by internal ballistics, the adjustable variables are the desired deflection, δ_L^* , and γ_L^* , the elevation angle. Values of these angles, together with prescriptions as to propulsion and fuzing (where any options exist) are furnished to the launcher crew as aiming instructions.

A standard missile, given the required $\vec{v}_L$ and subsequently flying through a vacuum, would arrive at a detonation point which may be called $\upsilon_{D(V)}$. The wind force prediction has resulted in the computation of an acceleration regime (excluding gravity, propulsion and control) during flight which produces a displacement measured by the vector from $\upsilon_{D(V)}$ to υ_D^* . The wind force prediction is that acceleration regime multiplied throughout by a mass of the standard projectile; it, of course, involves relative wind rather than true wind. The actual environment during flight, acting on a <u>standard</u> projectile, would produce still another detonation point $\upsilon_{D(A)}$ to υ_D^* is the wind force prediction error. It will be noted that this definition is not strictly one of wind "force". It includes the effect of the restoring moments ("weather-cock-stability") produced by wind resistance to a stable aerodynamic shape; these restoring moments in general impose a continuous variation in the coefficients of wind force.

The accuracy of wind force prediction depends, of course, on the observation methods used and the age of the meteorological information which is used for trajectory prediction. The magnitude of the error will generally decrease with increasing velocity, due to shorter time of exposure to the wind forces. In the same fashion the magnitude of the error will increase with range. However, simplifying assumptions, such as linearity with range, are risky because of the major impact errors accruing during low-speed portions of the trajectory.

8. The second component of launch accuracy is aiming accuracy. Aiming error is measured by the difference at launch initiation between the actual deflection and elevation δ_A and γ_A and those which would produce the required $\vec{v}_L$ (i.e., δ_L^* and γ_L^*). The aiming error in general has four components: 1) Sight reference error, 2) Sight reading error,
3) Boresight error, and 4) Firing time error.

The term "sight" is used in a general sense; the process of aiming always consists in setting azimuth and elevation angle differences between a 'bore line" and a "sight line", and then aligning the sight line on some reference line. This reference line may be established by an aiming stake, by compass and level bubble, by a gyroscope, or possibly other methods. The sight reference error is the deviation from a perfect reference line (in the e-axes system) of the sight line. The angle differences in azimuth and elevation can be neither set nor read with absolute precision; the error in the process of setting and reading these angles is the sight-reading error.

The 0 - 0 sight readings (when both deflection and elevation angles are set at zero) the sight line should have a fixed relation (usually parallel) to the bore line. The error in this fixed relation is the bore-sight error. The bore line, of course, is the longitudinal axis of a standard projectile mounted in the launcher and ready for launch.

The firing time error exists in the case of a non-stabilized moving launcher, and in the case of a moving target with non-continuous tracking. An airborne or shipborne launcher may be on a platform stabilized on the aiming reference line, in which case there is no firing time error. Otherwise provisions are made so that the launcher moves into proper alignment with the reference line, at which time the fire signal is given. The error in alignment at fire signal is the firing time error, and supersedes the sight reference error. In the case of a moving target, the launcher, whether fixed or moving, may track continuously applying a "lead", or it may lay on a fixed line and fire at a time calculated to intercept the target. In the second case, which is the less frequent for a fixed launcher, again a firing time error exists.

The magnitude of the aiming error can be much reduced by the quality of the system components involved, by frequent calibration, and by precise procedures. The effort to be invested in reduction of aiming error should be proportioned to the achievable gain in weapon system error, of which aiming error may be a minor component.

9. Launcher Error is to be distinguished from launch error, of which it is the final component. Launcher error develops between launch initiation (fire signal) and launcher clearance, and is the difference between the actual $\vec{v}_L$ and that which should have resulted for a standard projectile launched with the bore-line existent at launch initiation. (In most cases, the $\vec{v}_L$ which "should have resulted" has the same δ and γ as the bore-line at launch initiation, but this definition allows for some prediction of bore-line deviation during the internal ballistic phase.) Launcher error is a result of any non-standard characteristics or performance of the launcher (including its relation with ground surface or vehicle) and projectile as they interact during the internal ballistics phase, together with any unpredicted "tip-off error" due to cross-winds or the dynamics of bore alignment.

Launcher error cannot be further broken down without detailed description of the characteristics and design performance of a specific launcher-projectile combination. Launcher error may have some correlation with flight error. A non-standard characteristic of the projectile (e.g., a weight variation) may have results in internal ballistics and in free flight which tend to reinforce each other in their effect on weapon system error.

The launcher error is believed, however, to be independent of the other components of launch error. Thus, the variance of launch accuracy is the sum of the variances of trajectory prediction accuracy, aiming accuracy, and launcher accuracy.

10. The launch errors described above are common to all classes of projectiles enumerated in Para. 6. The categories of flight error which are relevant depend upon the class of projectile; thus, it is necessary to discuss each class in turn, starting with unguided ballistic projectiles. The only category of flight error to which this class of projectile is liable is termed trajectory performance error.

An unguided ballistic projectile during flight experiences accelerations due to two forces; 1) Gravity force, $\vec{F}_g$, and 2) Wind force, $\vec{F}_w$ (as defined in Paragraph 7). The planned trajectory has taken account of the forces exerted on a standard projectile by a predicted environment, and the deviation of a standard projectile in actual environment is already comprehended in the launch error (as trajectory prediction error). Thus, the difference in reactions of the standard and actual projectiles to the actual environment is of interest.

Since the force of gravity is not affected by characteristics of the projectile, only $\vec{F}_w$ need be considered for trajectory performance error. $\vec{F}_w$ acts on the center of pressure in the direction of the relative wind; its magnitude is expressed as $\frac{1}{2} C_R \, S \, \rho \, v^2$. C_R is the empirically determined resultant force coefficient which is dependent on the shape of the projectile and varies with the orientation of the body axes to the relative wind. S is the area of some reference, usually the cross-sectional area of the projectile body. Since $\vec{F}_w$ acts at the center of pressure, it creates a moment, $\vec{M}_w$, about the center of gravity. This moment leads to an angular acceleration which, for a stable projectile, tends to decrease C_R to its minimum. With regard to the orientation angles of $\vec{v}_w$, ξ and ζ, the behavior of the projectile is that of a damped spherical pendulum subjected to disturbing forces generated by the transverse component of $\vec{v}_w$. The magnitude of error will be dependent on the error in $\vec{v}_w$ and the errors in μ (the damping coefficient) and p (a coefficient of restoring moment). the quantities of μ and p are dependent on the following characteristics of the projectile: moment of inertia, center-of-gravity position, and the determinants of C_R - detailed shape and smoothness.

The trajectory performance error introduced by deviations in each of these characteristics can be estimated as a function of $\vec{\Delta v}_w$, the error in estimation of the true wind.

11. A programmed projectile differs from an unguided ballistic projectile by the addition of certain pre-programmed forces to $\vec{F}_w$ and $\vec{F}_g$. Each of the forces is programmed at a certain time and for a certain duration. These are designated by $\vec{F}_{C(P,i)}(t)$ or $\vec{F}_{T(P,i)}(t)$ with the subscript i available to distinguish among them if desired. They can, of course, be treated as one resultant $\vec{F}_P\,(t)$. Errors in these forces have an effect on the trajectory which is independent of that of the wind force except to the extent that they contribute to the disturbing force on the "spherical pendulum" and thus to the total axial acceleration. The effect on υ_D of errors in these forces is termed the programmed force error.

12. Corrected, guided, and homing projectiles all are subject to one or more corrections of trajectory during flight. These corrections compensate for some of the earlier errors, effectively removing them from accuracy calculations.

A corrected projectile corrects from the actual trajectory toward the planned trajectory; its guidance error supplants all components of launch and flight error except for the location error of the launcher or last point of up-date based on observations external to the missile.

A guided projectile does the same thing in principle, except last "up-date" is based on a target scene. Thus its only launch and flight error is the error of location of aim point in the target scene.

A homing projectile in principle is subject to no flight, launch, or target location errors, except possibly an aim point selection error due to the capabilities of the homing sensor. It can, of course, compensate for target motion, and it is the only class of projectile that can do so.

13. Since most projectiles are unguided during some early phase of their flight, the event $C1$ (first correction) establishes a division between the earlier categories of launch and flight accuracy and that of guidance accuracy. Guidance accuracy is in fact a probability density function (p.d.f.) of υ_D as a function of υ_{C1} (and other variables). If the p.d.f.'s for both υ_D and υ_{C1} are completely defined, the weapon system accuracy can be calculated by a multiple integral. In practice, this process is usually simplified by defining a "window" of υ_{C1} from which successful guidance is possible. The probability that υ_{C1} will lie within that window should then be regarded as the first of a series of factors that combine to form a "probability of good guidance", P_G.

Successful guidance depends upon the performance of a series of corrections $(C1, ...Ci, ...Cn)$. The number n of these corrections may be fixed in advance - as in the case where a number of "up-dates" are planned, based on known checkpoints en route. In other cases, the number n may be variable, dependent on the time spent on each "correction interval" and the time available between $C1$ and Cn.

In either case, a "correction interval" contains three events - an observation, a computation, and the application of a control force (possibly $\vec{O}$). The accuracies pertaining to each of these events combine to determine the p.d.f. of $\upsilon_{C(i+1)}$ as a function of υ_{Ci}. An n-fold multiple integration can be used to determine the total system accuracy; or, alternatively, the probability of passing through a set of n "windows" can be incorporated into P_G.

P_G has other factors, however. Given a reliability of 1.00, the necessary computations are made, the control force is applied, and the sensor making the observation functions within the specified limits. However, whether the observation is actually made also depends on the characteristics of the object or scene to be observed and the environment intervening between it and the observing sensor. The sensor must "recognize" the proper phenomenon before computations are initiated. The probability that such recognition occurs prior to each Ci is a factor in P_G. Suppose Oi is subscripted to denote the time of the i-th observation, and R_i is used to denote the probability that recognition occurs at that observation. Then there are two alternative forms for P_G:

1)
$$P_G = \underbrace{\int ....\int}_{(n\text{- fold})} R_i (d\upsilon)^n$$

where

$$R_i = R_i(\upsilon_{Oi})$$

2)
$$P_G = \prod_{i=1}^{n} P^*_{\upsilon_{Oi}} \; R_i$$

where $P^*_{\upsilon_{Oi}}$ denotes the probability that the projectile arrives within the "window" for successful performance of the i-th observation.

The first form is considered quite impractical for actual calculation.

14. The launch accuracy should then be stated as that p.d.f. of υ_D which is attained if $P_G = 1$ (according to Equation 2). P_G cannot equal unity unless the projectile arrives at the last "window" in a state such that a correction can be made (within the limits of the control system) which will bring the missile exactly to υ_D^* .

If $P_G = 1$ in the above terms, then the analysis of guidance accuracy is somewhat analogous to that of an unguided or programmed missile. The projectile is in effect "launched from a state υ_{Cn} and is subject to trajectory prediction error, trajectory performance error, and possible programmed force error for the remainder of the flight.

The "launcher location error" differs accordingly to the class of projectile being analyzed. For a corrected projectile, the observation On establishes a multi-dimensional vector $\upsilon_{Cn} - \upsilon_{Cn}^*$. This error vector must be combined with any error in the pre-launch establishment of υ_{Cn}^* with respect to υ_D^* to give the total "launcher location error.

For a guided projectile, the observation On is of υ_{Cn} with respect to υ_D^* although there may be some ambiguity in υ_D^* because of the resolution capability of the sensor. There is no error of υ_{Cn} with respect to υ_D^* since no υ_{Cn}^* was established pre-launch. A homing missile shares this characteristic, with the added advantage that υ_D^* is very nearly the optimum aim point - largely eliminating target location error.

As observed in Paragraph 7, computational errors for an unguided projectile can be reduced to a level which makes them insignificant compared to other errors. This is not necessarily true for guided projectiles, because of the limited time available for computation and the limited capability (and for analog systems, inherent inaccuracy) of the computer. In most cases the computation will be according to some simplified guidance law; each of these introduces a specific computational error.

In the case of a corrected missile, the wind force prediction relevant after Cn is the last portion of the wind force prediction used in pre-launch trajectory computation. In the case of a guided or homing projectile, the wind force prediction after Cn is normally $\vec{0}$. The wind force prediction error is, of course, the difference between the predicted wind force and the actual wind force experienced after Cn.

The trajectory performance error beyond Cn is identical in concept to that discussed earlier. It is measured by the difference in υ_D for a standard projectile flying through the actual environment and υ_D for the actual projectile flying through the actual environment.

If the projectile is programmed to perform any specific maneuvers after Cn (such as might be connected with optimum delivery of lethal mechanism, for instance), then programmed force errors, as discussed in Paragraph 11 may be included in the error occurring after Cn.

15. From the viewpoint set forth above, one of the factors most determinative of the accuracy of a guided projectile will be $\vec{r}_{Cn}$. Though certainly not linear with range, the errors of an unguided system are at least monotonically non-decreasing with range.

Of course, if Cn fails to be made for some reason, the missile may have a reasonable accuracy based on $Cn-1$ (failing that, on $Cn-2$, etc.). However, it seems quite reasonable to conjecture that if $Cn-1$ is successfully accomplished, Cn will be easier.

According to the thesis developed above, it is rather profitless to study the "guidance accuracy" of guided missiles. What is important is the Cn - the "probability of good guidance". Will the projectile, or its controller, actually recognize the check-point, aim point, or target at a location and time which will enable correction to enter a smaller "window" for the next subsequent observation? What is the probability that control force errors, combined with other flight errors, will throw it outside the next window? Granted $P_G = 1.00$, the accuracy can be very good indeed. However, one break in the chain of correction events probably leads to a distribution of υ_D no better than that for an unguided projectile.

Appendix A