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TORPEDO SURVIVAL IN MINE FIELD

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1.0 SUMMARY

The attached analysis derives an equation which expresses the probability of kill of a torpedo passing through a mine field. The equation is a function of:

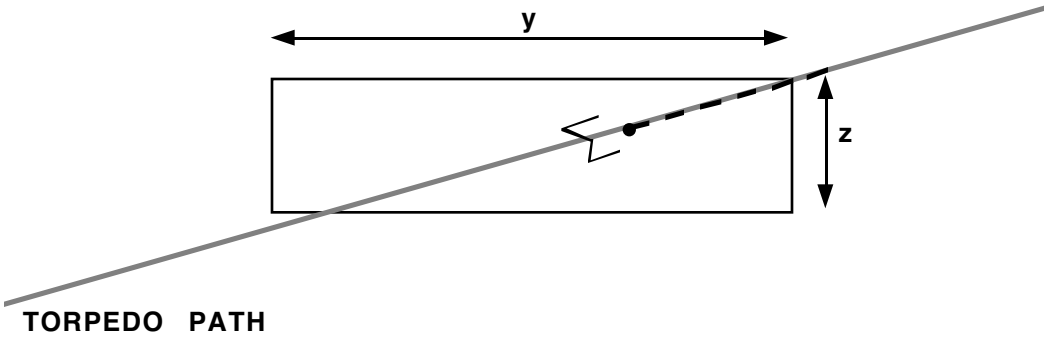
1. The uncertainty in locating the torpedo's path
2. The lethal radius of the mine
3. The number of mines

2.0 INTRODUCTION

The study proposes a weapon system concept which attacks a torpedo by placing a number of mines into an area through which the torpedo must pass. The mines hang by a chain attached to a float and detonate when the torpedo passes within the fuzing range. The analysis is simplified by several assumptions so that a single equation can be used to provide a rapid first order approximation.

3.0 ANALYSIS

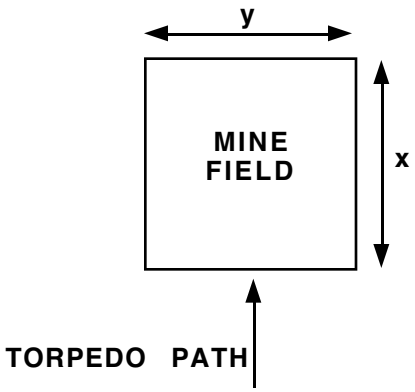
The torpedo path is assumed to be perpendicular to a rectangular region of uncertainty of dimension y and z as shown in the following figure:



Sketch 1

This region just touches the surface of the ocean and extends to a depth z below the surface.

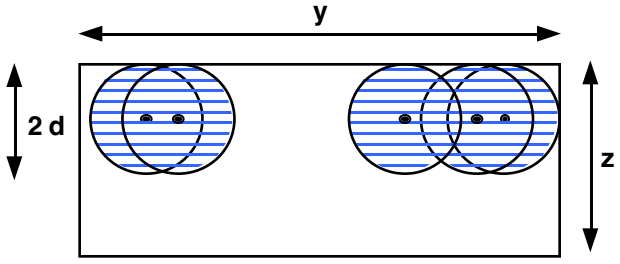
The mines are scattered in the path of the torpedo at some density ρ over a region which at least spans the width of the rectangle y. For this problem we shall assume that N mines are scattered uniformly in an area of width y and length $x = N/\rho y$ as shown in the plan view below:



Sketch 2

The mines hang by a chain at some depth d below the surface of the ocean. It is assumed that a torpedo passing within a distance d of a mine is killed.

Looking down the torpedo path into the mine field we see a picture like that shown below:



Sketch 3

The shaded regions are vulnerable zones for the torpedo. Ignoring edge effects, the probability of the torpedo surviving the first mine it encounter is.

$$p_s = 1 - \frac{\pi d^2}{3y}$$

If we assume that the mine effectiveness is designed to match the uncertainties in estimating the torpedo's path, we may set $z = 2d$ and the above equation becomes:

$$p_s = 1 - \frac{\pi d}{2y}$$

Assuming the placement of a mine to be statistically independent, the probability of torpedo kill from N mines is

$$p_k = 1 - \left(1 - \left[1 - \frac{\pi d}{2y}\right]^N\right)$$

If d is considerably less than y, we can approximate the above equation by

$$p_k = 1 - e^{-\frac{\pi d N}{2y}}$$

Appendix D